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<u>Exponential Functions</u> – $f(x) = a^x$, $a>0$, $a \neq 1$

- $x = n \in \mathbb{N}$, $a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ times}}$

$x = 0$, $a^0 = 1$, $a \neq 0$

$x = -n$, $a^{-n} = \frac{1}{a^n}$, $a \neq 0$

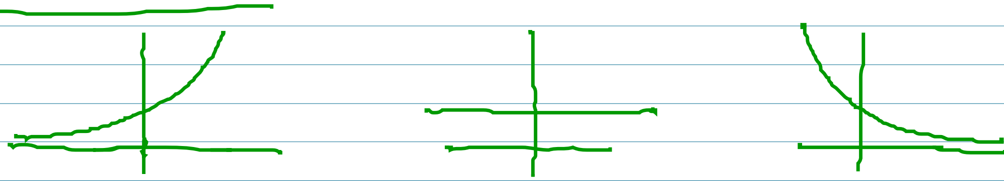
$x = \frac{p}{q} \in \mathbb{Q}$, $a^{\frac{p}{q}} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$

$p \in \mathbb{Z}, q \in \mathbb{N}$

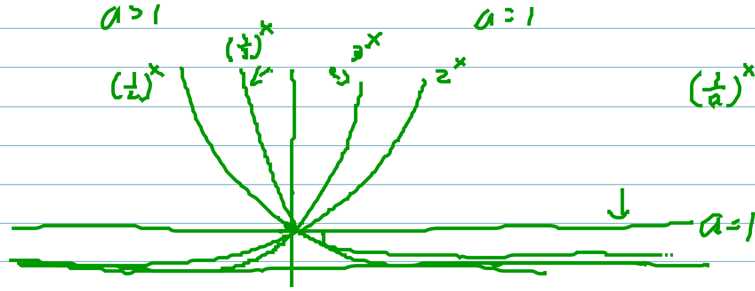
$x \in \mathbb{R} \setminus \mathbb{Q}$, Find $m, n \in \mathbb{Q}$ st $\overrightarrow{m} < x < \overleftarrow{n}$

and $\underrightarrow{a^m} < a^x < \underleftarrow{a^n}$

$- a^x = \lim_{r \to x} (a^r)$, $r \in \mathbb{Q}$

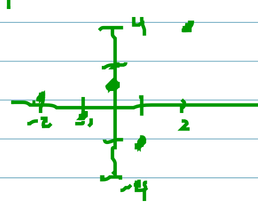


$a>1$ $\qquad$ $a=1$ $\qquad$ $0<a<1$



$(\frac{1}{a})^x = \frac{1}{a^x} = a^{-x}$

$a = -2$, $(-2)^x$



$x = \frac{1}{2} \to a^{1/2} = (-2)^{1/2}$
$= \sqrt{-2}$

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Exponential Functions – $f(x) = a^x$, $a > 0$, $a \neq 1$

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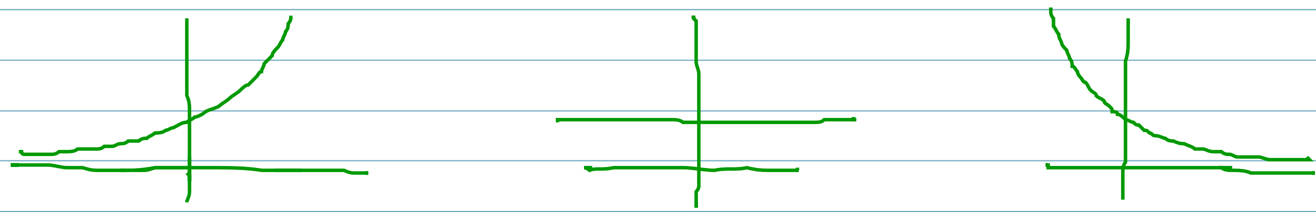
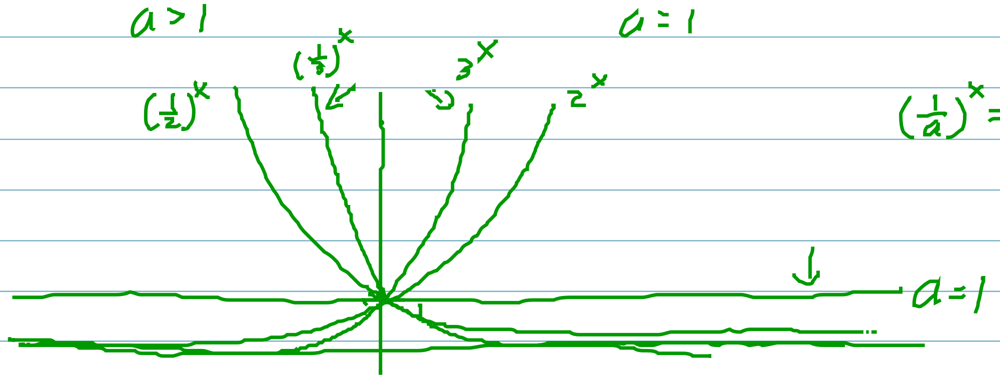


$a > 1$ $\quad$ $a = 1$ $\quad$ $0 < a < 1$



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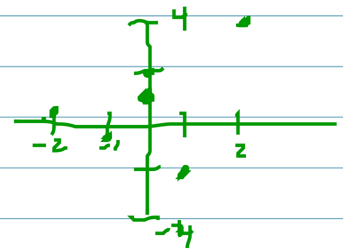

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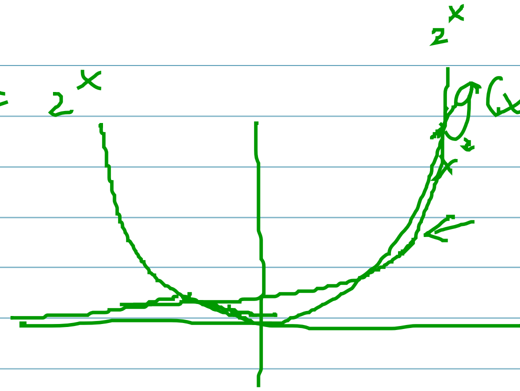
$f(x) = 2^x$ $\qquad$ $g(x) = x^2$



Laws of Exponents $\qquad a, b > 0, \quad x, y \in \mathbb{R}$

$a^x a^y = a^{x+y}$ $\qquad$ $\dfrac{a^x}{a^y} = a^{x-y} = \dfrac{1}{a^{y-x}}, \ a \neq 0$

$(a^x)^y = a^{xy}$ $\qquad$ $(ab)^x = a^x b^x$ $\qquad$ $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}, \ b \neq 0$

Theorem

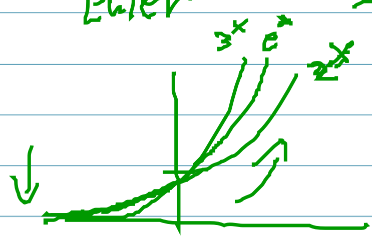
If $a > 0$, $a \neq 1$, Then $f(x) = a^x$ = continuous, Domain $= \mathbb{R}$, Range $= (0, \infty)$

$- \ a^x > 0, \ \forall x$ $\qquad$ HA - $y = 0$

Natural Number — $e = \lim_{n \to 0}\left(\underline{(1+n)^{1/n}}\right) = \lim_{n \to \infty}\left(\left(1+\frac{1}{n}\right)^n\right)$

Euler $\qquad \approx 2.71828182845904523536\ldots$

$f(x) = e^x$



$$f'(0) = \lim_{h \to 0}\left[\frac{f(0+h) - f(0)}{h}\right] = \lim_{h \to 0}\left(\frac{e^h - 1}{h}\right)$$

$$\lim_{h \to 0}\left(\frac{h}{h}\right)$$

$$\lim_{h \to 0}(1) = 1$$

$e \to (1+h)^{1/h}$

$e^h \to \left((1+h)^{1/h}\right)^h$

$e^h = 1 + h$

$e^h - 1 = h$