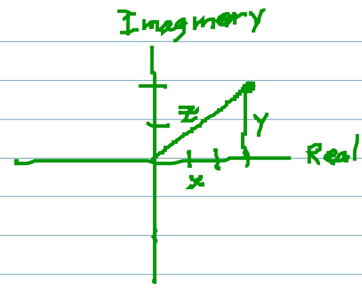
Complex Plane



$3+2i$

Real Imaginary

Complex Number – $z = x + yi$, $x, y \in \mathbb{R}$, $i = \sqrt{-1}$

- **Magnitude** – modulus
  – $|z| = |x+yi| = \sqrt{x^2+y^2} = \sqrt{z\bar{z}}$
- **conjugate** – $z = x + yi$
  $\bar{z} = x - yi$

Rectangular – $z = x + yi$

Polar – $z = r\cos(\theta) + r\sin(\theta)i = r(\cos(\theta) + \sin(\theta)i)$
$|z| = \sqrt{x^2+y^2} = r$

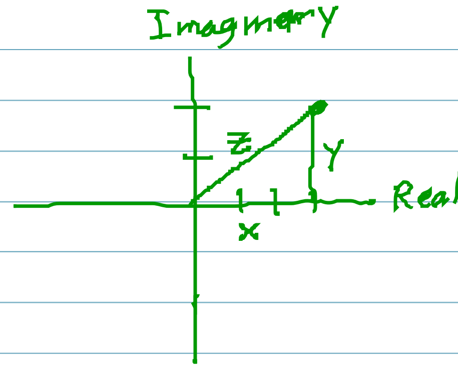
Theorem
If $z_1 = r_1(\cos(\theta_1) + i\sin(\theta_1))$ + $z_2 = r_2(\cos(\theta_2) + i\sin(\theta_2))$,
Then $z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right] \qquad z_2 \neq 0$$

DeMoivre's Theorem
If $z = r(\cos(\theta) + i\sin(\theta))$, Then $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

Complex Roots
Theorem
If $w = r(\cos(\theta_0) + i\sin(\theta_0)) \neq 0$, $n \in \mathbb{N}$ $n \geq 2$ Then
n distinct roots of w where $z_k = \sqrt[n]{r}\left(\cos\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right)$
$k = 0, 1, 2, \ldots, n-1$

Complex Plane



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Then $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right] \qquad z_2 \neq 0$$

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n distinct roots of w where $z_k = \sqrt[n]{r}\left(\cos\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta_0}{n} + \frac{2k\pi}{n}\right)\right)$

$k = 0, 1, 2, \ldots, n-1$

$$z = r(\cos(\theta) + i\sin(\theta))$$
$$= r e^{\theta i}$$

$$z^n = (r e^{\theta i})^n = r^n e^{n\theta i} = r^n(\cos(n\theta) + i\sin(n\theta))$$

$$\rightarrow \sqrt[n]{z} = \sqrt[n]{r e^{\theta i}} = (r e^{\theta i})^{1/n} = r^{1/n} e^{\frac{\theta}{n} i} = \sqrt[n]{r}\left(\cos\left(\tfrac{\theta}{n}\right) + i\sin\left(\tfrac{\theta}{n}\right)\right)$$

$$\rightarrow \vec{z}_K^{\,n} = \left[\sqrt[n]{r}\left(\cos\left(\tfrac{\theta_0}{n} + \tfrac{2k\pi}{n}\right) + i\sin\left(\tfrac{\theta_0}{n} + \tfrac{2k\pi}{n}\right)\right)\right]^n$$

$$= (\sqrt[n]{r})^n\left(\cos\left(n\left(\tfrac{\theta_0}{n} + \tfrac{2k\pi}{n}\right)\right) + i\sin\left(n\left(\tfrac{\theta_0}{n} + \tfrac{2k\pi}{n}\right)\right)\right)$$

$$= |r|(\cos(\theta_0 + 2k\pi) + i\sin(\theta_0 + 2k\pi))$$

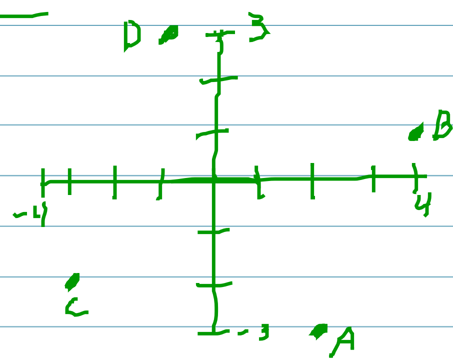
$$= r(\cos(\theta_0) + i\sin(\theta_0))$$

$A = 2 - 3i$

$B = 4 + i$

$C = -3 - 2i$

$D = -1 + 3i$



$$z = -1 + i \qquad r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$$
$$= \sqrt{2}\left(\cos\left(\tfrac{3\pi}{4}\right) + i\sin\left(\tfrac{3\pi}{4}\right)\right) \qquad \theta_R = \tan^{-1}\left(\tfrac{1}{-1}\right) = -\tfrac{\pi}{4} \rightarrow \tfrac{3\pi}{4}$$

$$2\left(\cos\left(\tfrac{5\pi}{6}\right) + i\sin\left(\tfrac{5\pi}{6}\right)\right)$$
$$2\left(-\tfrac{\sqrt{3}}{2} + i\,\tfrac{1}{2}\right)$$
$$-\sqrt{3} + i$$

$$z = 2(\cos(80^\circ) + i\sin(80^\circ)) \qquad w = 6(\cos(200^\circ) + i\sin(200^\circ))$$

$$wz = 12(\cos(280^\circ) + i\sin(280^\circ))$$
$$\frac{w}{z} = 3(\cos(120^\circ) + i\sin(120^\circ)) = 3\left(-\tfrac{1}{2} + i\left(\tfrac{\sqrt{3}}{2}\right)\right) = -\tfrac{3}{2} + \tfrac{3\sqrt{3}}{2} i$$