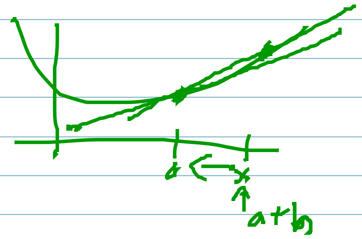
Tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

~~$m = \lim_{h \to 0} \left(f(x+h) - f(x) \right)$~~ $\quad m = \lim_{x \to a} \left(\dfrac{f(x) - f(a)}{x - a} \right)$ if it exists



$$m = \lim_{h \to 0} \left(\frac{f(a+h) - f(a)}{h} \right)$$

Velocity – $v(a) = \lim_{h \to 0} \left(\dfrac{f(a+h) - f(a)}{h} \right)$ position

Derivative – $f'(a) = \lim_{h \to 0} \left(\dfrac{f(a+h) - f(a)}{h} \right)$

$$= \lim_{x \to a} \left(\frac{f(x) - f(a)}{x - a} \right)$$

Rates of Change

Increment – $\Delta x = x_2 - x_1 \qquad \Delta y = y_2 - y_1 = f(x_2) - f(x_1)$

Average rate of change of y with respect to x

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Instantaneous rate of change of y with respect to x

$$\lim_{\Delta x \to 0} \left(\frac{\Delta y}{\Delta x} \right) = \lim_{x_2 \to x_1} \left(\frac{f(x_2) - f(x_1)}{x_2 - x_1} \right) = f'(x_1)$$

$f'(a)$ is instantaneous rate of change of $y = f(x)$ with respect to x when $x = a$

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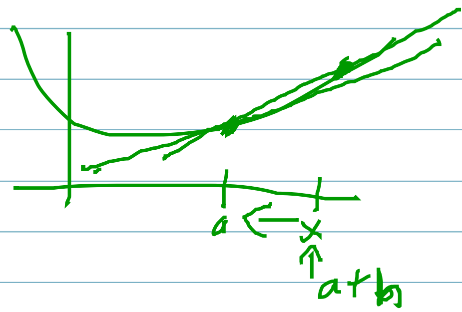
$$\Uparrow$$

$$m = \lim_{h \to 0} \left(\frac{f(a+h) - f(a)}{h} \right)$$



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Speed $= |v(a)| = |f'(a)|$

$f(x) = 7x - 5$

$$f'(a) = \lim_{h\to 0}\left(\frac{f(a+h) - f(a)}{h}\right) = \lim_{h\to 0}\left(\frac{[7(a+h) - 5] - (7a - 5)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\cancel{7a} + 7h \cancel{- 5} \cancel{- 7a} \cancel{+ 5}}{h}\right) = \lim_{h\to 0}\left(\frac{7\cancel{h}}{\cancel{h}}\right) = \lim_{h\to 0}(7) = 7$$

$f(x) = 5x^2 - 3x + 9$

$a^2 + 2ah + h^2$

$$f'(a) = \lim_{h\to 0}\left(\frac{f(a+h) - f(a)}{h}\right) = \lim_{h\to 0}\left(\frac{5(a+h)^2 - 3(a+h) + 9 - (5a^2 - 3a + 9)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\cancel{5a^2} + 10ah + 5h^2 \cancel{- 3a} - 3h \cancel{+ 9} \cancel{- 5a^2} \cancel{+ 3a} \cancel{- 9}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{10ah + 5h^2 - 3h}{h}\right) = \lim_{h\to 0}\left(\frac{\cancel{h}(10a + 5h - 3)}{\cancel{h}}\right) = 10a - 3$$

$f(x) = \frac{3}{\sqrt{x-2}}$

$$f'(a) = \lim_{h\to 0}\left(\frac{\frac{3}{\sqrt{a+h-2}} - \frac{3}{\sqrt{a-2}}}{h}\right) = \lim_{h\to 0}\left(\frac{3\sqrt{a-2} - 3\sqrt{a+h-2}}{h\sqrt{a+h-2}\sqrt{a-2}}\right)\frac{(3\sqrt{a-2} + 3\sqrt{a+h-2})}{(3\sqrt{a-2} + 3\sqrt{a+h-2})}$$

$$= \lim_{h\to 0}\left(\frac{9(a-2) - 9(a+h-2)}{h\sqrt{a+h-2}\sqrt{a-2}\,(3\sqrt{a-2} + 3\sqrt{a+h-2})}\right) = \lim_{h\to 0}\left(\frac{\cancel{9a} \cancel{- 18} \cancel{- 9a} - 9h \cancel{+ 18}}{h\sqrt{a+h-2}\sqrt{a-2}\,(3\sqrt{a-2} + 3\sqrt{a+h-2})}\right)$$

$$= \lim_{h\to 0}\left(\frac{-9\cancel{h}}{\cancel{h}\sqrt{a+h-2}\sqrt{a-2}\,(3\sqrt{a-2} + 3\sqrt{a+h-2})}\right) = \frac{-9}{\sqrt{a-2}\sqrt{a-2}\,(3\sqrt{a-2} + 3\sqrt{a-2})}$$

$$= \frac{-9}{(a-2)\,6\sqrt{a-2}} = \frac{-3}{2(a-2)\sqrt{a-2}} = \frac{-3}{2(a-2)^{3/2}}$$