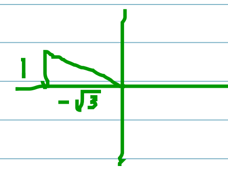
$\tan\left(\frac{19\pi}{12}\right) = \tan\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)$

$\rightarrow \frac{\pi}{6} \quad \frac{\pi}{4} \quad \frac{\pi}{3} \quad \frac{\pi}{2} \quad \frac{2\pi}{3} \quad \frac{3\pi}{4} \quad \frac{5\pi}{6} \quad \pi$

$\frac{2\pi}{12} \quad \frac{3\pi}{12} \quad \frac{4\pi}{12} \quad \frac{6\pi}{12} \quad \frac{8\pi}{12} \quad \frac{9\pi}{12} \quad \frac{10\pi}{12} \quad \frac{12\pi}{12}$

$= \dfrac{\tan\left(\frac{3\pi}{4}\right) + \tan\left(\frac{5\pi}{6}\right)}{1 - \tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{5\pi}{6}\right)}$



$= \dfrac{-1 + \left(-\frac{\sqrt{3}}{3}\right)}{1 - (+1)\left(+\frac{\sqrt{3}}{3}\right)} \cdot \dfrac{3}{3} = \dfrac{-3 - \sqrt{3}}{3 - \sqrt{3}}$

$= \dfrac{-1 - \frac{1}{\sqrt{3}}}{1 - (+1)\left(+\frac{1}{\sqrt{3}}\right)} = \dfrac{-\sqrt{3} - 1}{\sqrt{3} - 1} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$\sec\left(-\frac{\pi}{12}\right) = \sec\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \dfrac{4}{\sqrt{6}+\sqrt{2}} \cdot \dfrac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \dfrac{4(\sqrt{6}-\sqrt{2})}{36-4} = \dfrac{\sqrt{6}-\sqrt{2}}{8}$

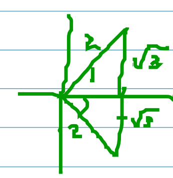
$\cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \dfrac{\sqrt{6} + \sqrt{2}}{4}$

$\cos\left(\frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right) + \sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{12} - \frac{5\pi}{12}\right) = \cos\left(\frac{-4\pi}{12}\right)$

$= \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$



$\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{5\pi}{18}\right) + \cos\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right) = \sin\left(\frac{\pi}{18} + \frac{5\pi}{18}\right) = \sin\left(\frac{6\pi}{18}\right) = \sin\left(\frac{\pi}{3}\right)$

$= \frac{\sqrt{3}}{2}$

$\cos(\alpha) = \frac{\sqrt{5}}{5}$, $\alpha \in \left(0, \frac{\pi}{2}\right)$

$\sin(\alpha) = \frac{2\sqrt{5}}{5}$

$\sin(\beta) = -\frac{4}{5}$, $\beta \in \left(-\frac{\pi}{2}, 0\right)$

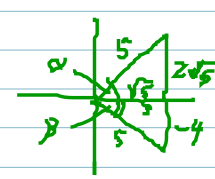
$\cos(\beta) = \frac{3}{5}$

$y^2 + \sqrt{5}^2 = 5^2$

$y^2 + 5 = 25$

$y^2 = 20$

$y = 2\sqrt{5}$



$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$= \left(\frac{2\sqrt{5}}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{\sqrt{5}}{5}\right)\left(-\frac{4}{5}\right)$

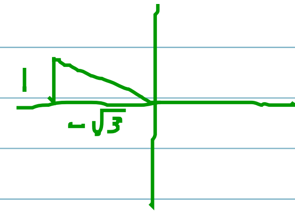
$= \dfrac{6\sqrt{5} - 4\sqrt{5}}{25} = \dfrac{2\sqrt{5}}{25}$

$$\tan\left(\frac{19\pi}{12}\right) = \tan\left(\frac{3\pi}{4} + \frac{5\pi}{6}\right)$$

$\rightarrow \frac{\pi}{6} \quad \frac{\pi}{4} \quad \frac{\pi}{3} \quad \frac{\pi}{2} \quad \frac{2\pi}{3} \quad \frac{3\pi}{4} \quad \frac{5\pi}{6} \quad \pi$

$\frac{2\pi}{12} \quad \frac{3\pi}{12} \quad \frac{4\pi}{12} \quad \frac{6\pi}{12} \quad \frac{8\pi}{12} \quad \frac{9\pi}{12} \quad \frac{10\pi}{12} \quad \frac{12\pi}{12}$

$$= \frac{\tan\left(\frac{3\pi}{4}\right) + \tan\left(\frac{5\pi}{6}\right)}{1 - \tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{5\pi}{6}\right)}$$



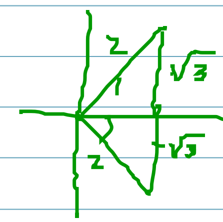
$$= \frac{-1 + \left(-\frac{\sqrt{3}}{3}\right)}{1 - (+1)\left(+\frac{\sqrt{3}}{3}\right)} \cdot \frac{3}{3} = \frac{-3-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{-1 - \frac{1}{\sqrt{3}}}{1-(+1)\left(+\frac{1}{\sqrt{3}}\right)} = \frac{-\sqrt{3}-1}{\sqrt{3}-1} \quad \frac{\sqrt{3}}{\sqrt{3}}$$

$$\sec\left(-\frac{\pi}{12}\right) = \sec\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \frac{4}{\sqrt{6}+\sqrt{2}} \quad \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{4(\sqrt{6}-\sqrt{2})}{36-4} = \frac{\sqrt{6}-\sqrt{2}}{8}$$

$$\cos\left(\frac{\pi}{6} - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$
$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{6}+\sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right) + \sin\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{12} - \frac{5\pi}{12}\right) = \cos\left(\frac{-4\pi}{12}\right)$$
$$= \cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$



$$\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{5\pi}{18}\right) + \cos\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right) = \sin\left(\frac{\pi}{18} + \frac{5\pi}{18}\right) = \sin\left(\frac{6\pi}{18}\right) = \sin\left(\frac{\pi}{3}\right)$$
$$= \frac{\sqrt{3}}{2}$$

$\cos(\alpha) = \frac{\sqrt{5}}{5}$, $\alpha \in \left[0, \frac{\pi}{2}\right)$ $\qquad$ $\sin(\beta) = -\frac{4}{5}$, $\beta \in \left(-\frac{\pi}{2}, 0\right)$

$\sin(\alpha) = \frac{2\sqrt{5}}{5}$ $\qquad$ $\cos(\beta) = \frac{3}{5}$

$y^2 + \sqrt{5}^2 = 5^2$

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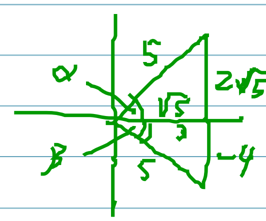


$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$= \left(\frac{2\sqrt{5}}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{\sqrt{5}}{5}\right)\left(-\frac{4}{5}\right)$$
$$= \frac{6\sqrt{5} - 4\sqrt{5}}{25} = \frac{2\sqrt{5}}{25}$$

$$\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$
$$= \left(\frac{2\sqrt{5}}{5}\right)\left(\frac{3}{5}\right) - \left(\frac{\sqrt{5}}{5}\right)\left(-\frac{4}{5}\right) = \frac{6\sqrt{5}+4\sqrt{5}}{25} = \frac{10\sqrt{5}}{25} = \frac{2\sqrt{5}}{5}$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$= \left(\frac{\sqrt{5}}{5}\right)\left(\frac{3}{5}\right) - \left(\frac{2\sqrt{5}}{5}\right)\left(-\frac{4}{5}\right) = \frac{3\sqrt{5}+8\sqrt{5}}{25} = \frac{11\sqrt{5}}{25}$$

$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$
$$= \left(\frac{\sqrt{5}}{5}\right)\left(\frac{3}{5}\right) + \left(\frac{2\sqrt{5}}{5}\right)\left(-\frac{4}{5}\right) = \frac{3\sqrt{5}-8\sqrt{5}}{25} = \frac{-5\sqrt{5}}{25} = -\frac{\sqrt{5}}{5}$$

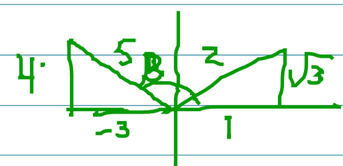
$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\frac{2\sqrt{5}}{25}}{\frac{11\sqrt{5}}{25}} = \frac{2\sqrt{5}}{25}\,\frac{25}{11\sqrt{5}} = \frac{2}{11}$$

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\frac{2\sqrt{5}}{5}}{-\frac{\sqrt{5}}{5}} = \frac{2\sqrt{5}}{5}\left(\frac{-5}{\sqrt{5}}\right) = = -2$$

$\tan(\alpha) = -\frac{4}{3}$, $\alpha \in \left(\frac{\pi}{2}, \pi\right)$ $\qquad$ $\cos(\beta) = \frac{1}{2}$, $\beta \in \left(0, \frac{\pi}{2}\right)$

$\sin(\alpha) = \frac{4}{5}$ $\qquad$ $\sin(\beta) = \frac{\sqrt{3}}{2}$

$\cos(\alpha) = -\frac{3}{5}$



$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$
$$= \left(\frac{4}{5}\right)\left(\frac{1}{2}\right) + \left(-\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right) \leftarrow$$
$$= \frac{4-3\sqrt{3}}{10} \leftarrow$$

$\cancel{\sin\left(\frac{4}{5}\right)\cos\left(\frac{1}{2}\right) + \cos\left(-\frac{3}{5}\right)\sin\left(\frac{\sqrt{3}}{2}\right)}$

$$\sin(\alpha-\beta) = \left(\frac{4}{5}\right)\left(\frac{1}{2}\right) - \left(-\frac{3}{5}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{4+3\sqrt{3}}{10}$$

$$\cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$
$$= \left(-\frac{3}{5}\right)\left(\frac{1}{2}\right) - \left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{-3-4\sqrt{3}}{10}$$

$$\cos(\alpha-\beta) = \left(-\frac{3}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{4}{5}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{-3+4\sqrt{3}}{10}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\frac{4-3\sqrt{3}}{10}}{\frac{-3-4\sqrt{3}}{10}} = \frac{4-3\sqrt{3}}{-3-4\sqrt{3}}$$

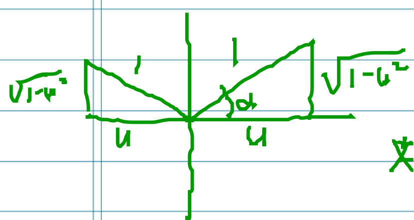
$$\tan(\alpha-\beta) = \frac{\frac{4+3\sqrt{3}}{10}}{\frac{-3+4\sqrt{3}}{10}} = \frac{4+3\sqrt{3}}{-3+4\sqrt{3}}$$

$$\sin\left(\tfrac{\pi}{2}-\theta\right) = \cos(\theta)$$

$$\sin\left(\tfrac{\pi}{2}\right)\cos(\theta) - \cos\left(\tfrac{\pi}{2}\right)\sin(\theta)$$

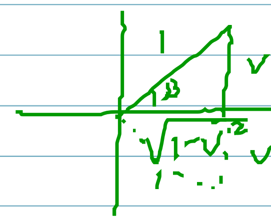
$$\underbrace{(1)\cos(\theta)}_{\cos(\theta)} - \underbrace{(0)\sin(\theta)}$$

$$\cos(\underbrace{\cos^{-1}(u)}_{\alpha} + \underbrace{\sin^{-1}(v)}_{\beta}) = \cos(\underbrace{\cos^{-1}(u)}_{\alpha})\cos(\underbrace{\sin^{-1}(v)}_{\beta}) - \sin(\underbrace{\cos^{-1}(u)}_{\alpha})\sin(\underbrace{\sin^{-1}(v)}_{\beta})$$



$$1^2 = u^2 + y^2$$
$$1 - u^2 = y^2$$
$$\cancel{\pm}\sqrt{1-u^2} = y$$

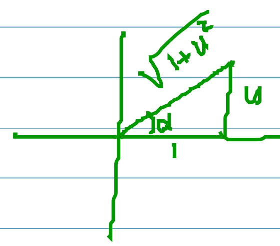
$$1^2 = v^2 + x^2$$
$$1 - v^2 = x^2$$
$$\cancel{\pm}\sqrt{1-v^2} = x$$

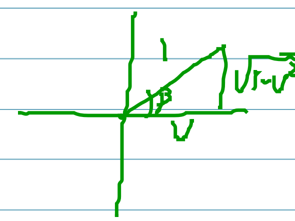


$$\cos(\alpha) = u \qquad \sin(\beta) = v$$
$$\sin(\alpha) = \sqrt{1-u^2} \qquad \cos(\beta) = \sqrt{1-v^2}$$

$$\cos(\cos^{-1}(u) + \sin^{-1}(v)) = u\sqrt{1-v^2} - \sqrt{1-u^2}\, v$$

$$\sin(\underbrace{\tan^{-1}(u)}_{\alpha} + \underbrace{\cos^{-1}(v)}_{\beta})$$



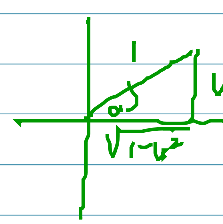


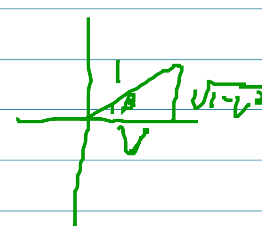
$$\sin(\alpha) = \frac{u}{\sqrt{1-u^2}} \qquad \sin(\beta) = \sqrt{1-v^2}$$
$$\cos(\alpha) = \frac{1}{\sqrt{1-u^2}} \qquad \cos(\beta) = v$$

$$\sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) = \left(\frac{u}{\sqrt{1-u^2}}\right)v + \left(\frac{1}{\sqrt{1-u^2}}\right)\sqrt{1-v^2}$$

$$= \frac{uv + \sqrt{1-v^2}}{\sqrt{1-u^2}}$$

$$\tan(\underbrace{\sin^{-1}(u)}_{\alpha} - \underbrace{\cos^{-1}(v)}_{\beta})$$





$$\tan(\alpha) = \frac{u}{\sqrt{1-u^2}} \qquad \tan(\beta) = \frac{\sqrt{1-v^2}}{v}$$

$$\tan(\alpha-\beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)} = \frac{\frac{v}{v}\frac{u}{\sqrt{1-u^2}} - \frac{\sqrt{1-v^2}}{v}\frac{\sqrt{1-u^2}}{\sqrt{1-u^2}}}{\to 1 + \frac{u}{\sqrt{1-u^2}}\frac{\sqrt{1-v^2}}{v}}$$

$$\frac{\dfrac{uv - \sqrt{1-v^2}\sqrt{1-u^2}}{\to v\sqrt{1-u^2}}}{\dfrac{v\sqrt{1-u^2} + u\sqrt{1-v^2}}{\to v\sqrt{1-u^2}}}$$

$$= \boxed{\frac{uv - \sqrt{1-v^2}\sqrt{1-u^2}}{v\sqrt{1-u^2} + u\sqrt{1-v^2}}}$$