$f'(x) =$ slope of $f(x)$

Increasing/Decreasing Test

If $f'(x) > 0$, $\forall x \in I$: Interval, Then $f(x)$ = increasing, $\forall x \in I$
If $f'(x) < 0$, $\forall x \in I$: Interval, Then $f(x)$ = decreasing, $\forall x \in I$

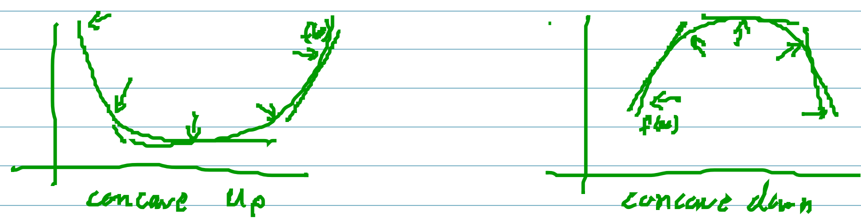
First Derivative Test – f = continuous, c = Critical Number

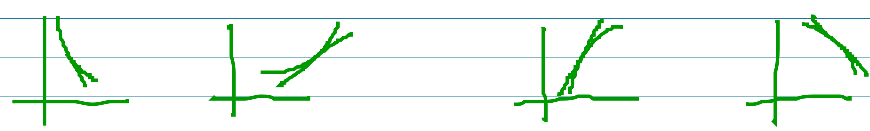
↗↘ If f' changes from $+ \to -$ at c, Then Local Maximum at c
↘↗ If f' changes from $- \to +$ at c, Then Local Minimum at c
If f' does not change sign at c, Then NO Local Min/Max at c

$f''(x) =$ slope of $f'(x)$ = slope of slope of $f(x)$ = concavity of $f(x)$

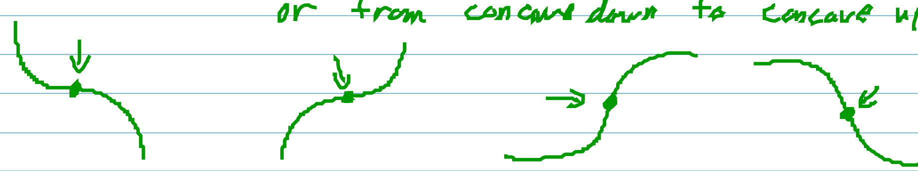
concave up – graph of $f(x)$ is ABOVE graph of tangent on I

concave down – graph of $f(x)$ is BELOW graph of tangent on I





Inflection Point – $P = (x, y)$ st f = continuous at P and change from concave up to concave down or from concave down to concave up at P



$f'(x)$ = slope of $f(x)$

Increasing/Decreasing Test

If $f'(x) > 0$, $\forall x \in I$: Interval, Then $f(x)$ = increasing, $\forall x \in I$
If $f'(x) < 0$, $\forall x \in I$: Interval, Then $f(x)$ = decreasing, $\forall x \in I$

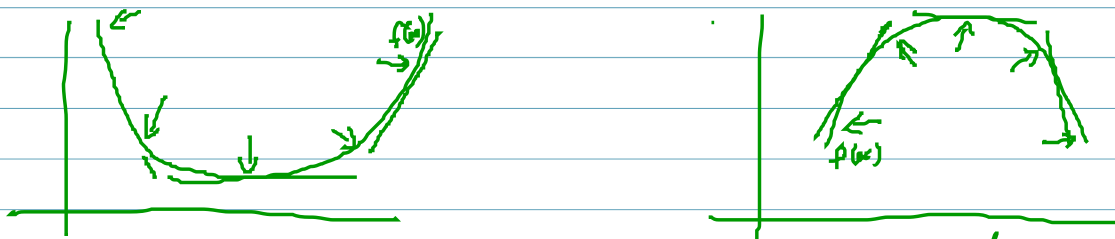
First Derivative Test – f = continuous, c = Critical Number

If f' changes from $+ \to -$ at c, Then Local Maximum at c
If f' changes from $- \to +$ at c, Then Local Minimum at c
If f' does not change sign at c, Then NO Local Min/Max at c

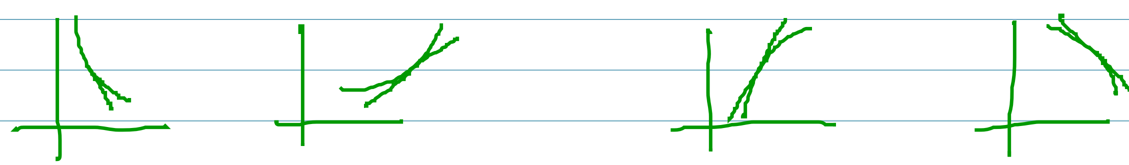
$f''(x)$ – slope of $f'(x)$ = slope of slope of $f(x)$ = concavity of $f(x)$

concave up – graph of $f(x)$ is ABOVE graph of tangent on I

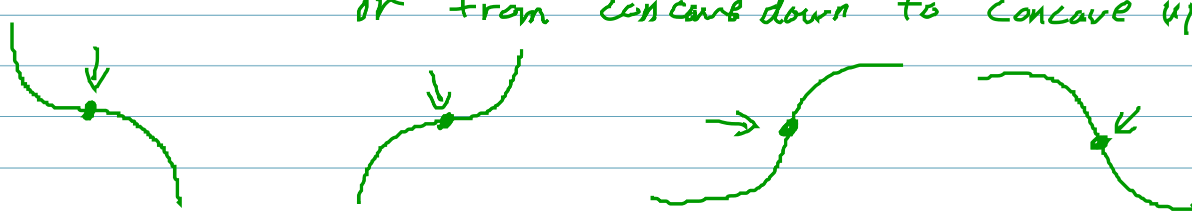
concave down – graph of $f(x)$ is BELOW graph of tangent on I



concave up   concave down



Inflection Point – $P = (x,y)$ st f = continuous at P and change from concave up to concave down or from concave down to concave up at P



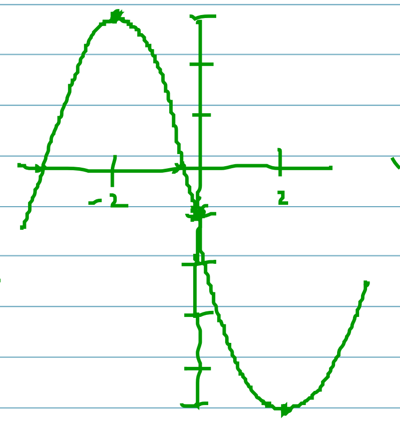
Concavity Test

If $f''(x) > 0, \forall x \in I$ = Interval, then $f(x)$ = concave up, $\forall x \in I$
If $f''(x) < 0, \forall x \in I$ = Interval, Then $f(x)$ = concave down, $\forall x \in I$

2nd Derivative Test – f'' = continuous near c

If $f'(c) = 0$, $f''(c) > 0$, Then Local Minimum at c
If $f'(c) = 0$, $f''(c) < 0$, Then Local Maximum at c

$f(x) = x^3 - 12x - 4$
$f'(x) = 3x^2 - 12$
$f''(x) = 6x$



$3x^2 - 12 = 0$
$3(x^2 - 4) =$
$3(x+2)(x-2) =$
$x = \pm 2$

$6x = 0$
$x = 0$

x	-2		0		2
f(x)	12		-4		-20
f'(x)	+ ↗ 0	↘	—	↘	0 ↗ +
f''(x)	− Max		0		+ Min

$f'(-3) = 3(-3)^2 - 12 = 15$
$f'(0) = 3(0)^2 - 12 = -12$
$f'(3) = 3(3)^2 - 12 = 15$

$f''(-2) = 6(-2) = -12$
$f''(2) = 6(2) = 12$

$f(-2) = (-2)^3 - 12(-2) - 4 = -8 + 24 - 4 = 12$
$f(0) = (0)^3 - 12(0) - 4 = -4$
$f(2) = (2)^3 - 12(2) - 4 = 8 - 24 - 4 = -20$

Local Max – $(-2, 12)$
Local Min – $(2, -20)$
Inflection Point – $(0, -4)$

Increase – $(-\infty, -2) \cup (2, \infty)$
Decrease – $(-2, 2)$

Concave Up – $(0, \infty)$
Concave down – $(-\infty, 0)$