

Area -

$A = lw$ $A = \frac{1}{2}bh$ $A = A_1 + A_2 + A_3$ $A = ?$

$n=6$

Midpoint Left Right

Sigma ~~Notation~~ Notation

$$\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n$$

$$\sum_{i=1}^5 (2i+1) = (2(1)+1) + (2(2)+1) + (2(3)+1) + (2(4)+1) + (2(5)+1)$$

$$= 3 + 5 + 7 + 9 + 11 = 27$$

$$R_n = \sum_{i=1}^n f(x_i) \Delta x_i$$

$$L_n = \sum_{i=1}^n f(x_{i-1}) \Delta x_i$$

$$M_n = \sum_{i=1}^n f\left(\frac{x_{i-1} + x_i}{2}\right) \Delta x_i$$

$\Delta x = x_n - x_{n-1}$
 $\Delta x_n = \frac{b-a}{n}$

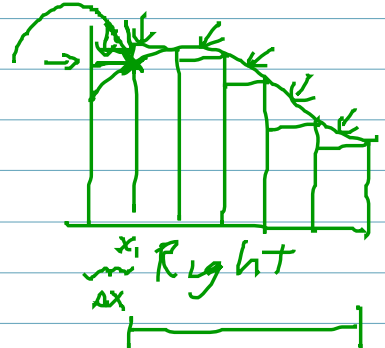
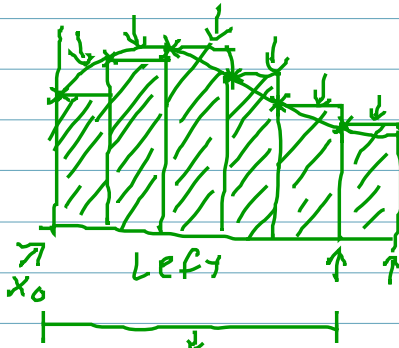
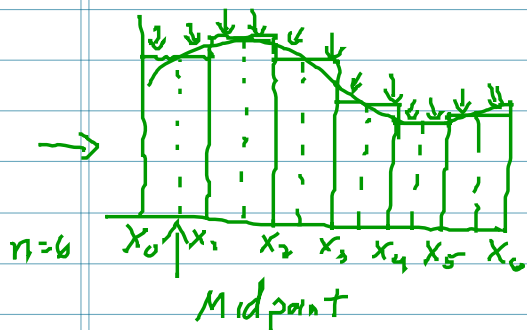
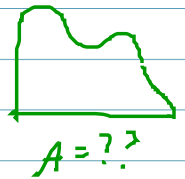
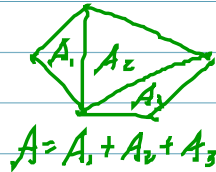
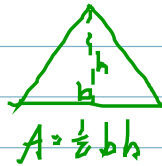
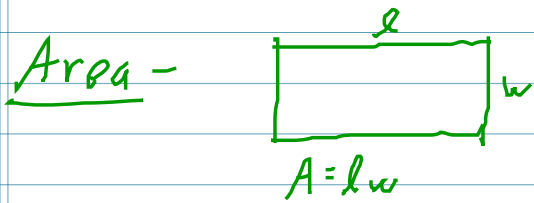
$A = \lim_{n \rightarrow \infty} (R_n) = \lim_{n \rightarrow \infty} (L_n) = \lim_{n \rightarrow \infty} (M_n)$

choose any small ϵ , can find N large enough st

$$\forall n \geq N, |A - R_n| < \epsilon, |A - L_n| < \epsilon, |A - M_n| < \epsilon$$

sample point - x_i

$x_0 = a$
 $x_1 = a + \Delta x$
 $x_2 = a + 2\Delta x$ \rightarrow ~~x_i~~ $x_i = a + i\Delta x$ $x_i \neq x_{i-1}$



Sigma ~~Notation~~ Notation

$$\sum_{i=m}^n a_i = a_m + a_{m+1} + a_{m+2} + \dots + a_n$$

$$\sum_{i=3}^5 (2i+1) = (2(3)+1) + (2(4)+1) + (2(5)+1) = 7 + 9 + 11 = 27$$

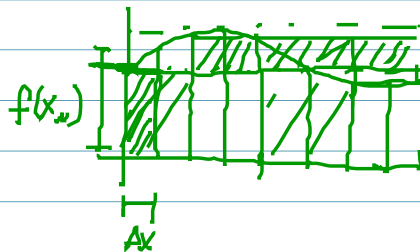
$$R_n = \sum_{i=1}^n f(x_i) \Delta x_n$$

$$L_n = \sum_{i=1}^n f(x_{i-1}) \Delta x_n$$

$$M_n = \sum_{i=1}^n f\left(\frac{x_i + x_{i-1}}{2}\right) \Delta x_n$$

$$\Delta x = x_n - x_{n-1}$$

$$\Delta x_n = \frac{b-a}{n}$$

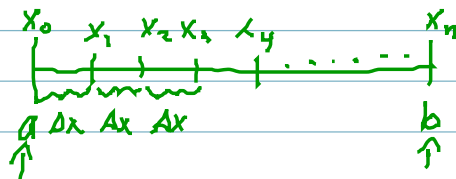


$$A = \lim_{n \rightarrow \infty} (R_n) = \lim_{n \rightarrow \infty} (L_n) = \lim_{n \rightarrow \infty} (M_n)$$

choose any small ϵ , can find N large enough st

$$\forall n \geq N, |A - R_n| < \epsilon, |A - L_n| < \epsilon, |A - M_n| < \epsilon$$

Sample point - x_i



$$x_0 = a$$

$$x_1 = a + \Delta x$$

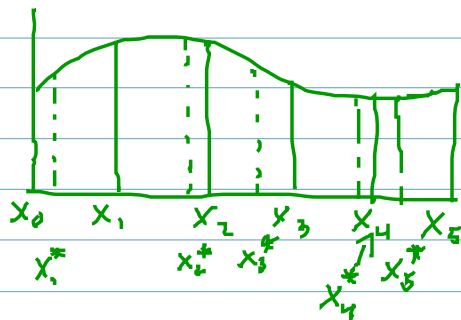
$$x_2 = a + 2\Delta x$$

$$\rightarrow \cancel{x_i} \quad x_i = a + i\Delta x$$

$$\downarrow \quad \downarrow$$

$$x_i \neq x_{i-1}$$

$$A = \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(x_i^*) \Delta x_i \right), \quad x_i^* \in (x_{i-1}, x_i)$$



$$R_4 \quad f(x) = x^2 + 2 \quad [-1, 1]$$

$$\Delta x_i = \frac{1 - (-1)}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\begin{aligned} \Rightarrow x_0 &= -1 \\ \rightarrow x_1 &= -\frac{1}{2} \\ \rightarrow x_2 &= 0 \\ \rightarrow x_3 &= \frac{1}{2} \\ \rightarrow x_4 &= 1 \end{aligned}$$

$$\begin{aligned} f(x_0) &= 3 \\ f(x_1) &= \frac{9}{4} \\ f(x_2) &= 2 \\ f(x_3) &= \frac{9}{4} \\ f(x_4) &= 3 \end{aligned}$$

$$R_4 = \sum_{i=1}^4 f(x_i^*) \Delta x$$

$$= f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x + f(x_4) \Delta x$$

$$= \frac{1}{2} \left(\frac{9}{4} + 2 + \frac{9}{4} + 3 \right)$$

$$= \frac{1}{2} \left(5 + \frac{18}{4} \right) = \frac{1}{2} \left(\frac{38}{4} \right) = \frac{19}{4}$$

$$L_4 = \sum_{i=1}^4 f(x_{i-1}) \Delta x = f(x_0) \Delta x + f(x_1) \Delta x + f(x_2) \Delta x + f(x_3) \Delta x$$

$$= \frac{1}{2} \left(3 + \frac{9}{4} + 2 + \frac{9}{4} \right) = \frac{19}{4}$$

$$\begin{aligned} x_1^* &= -\frac{3}{4} \\ x_2^* &= -\frac{1}{4} \\ x_3^* &= \frac{1}{4} \\ x_4^* &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} f(x_1^*) &= \frac{41}{16} \\ f(x_2^*) &= \frac{33}{16} \\ f(x_3^*) &= \frac{33}{16} \\ f(x_4^*) &= \frac{41}{16} \end{aligned}$$

$$M_4 = \sum_{i=1}^4 f\left(\frac{x_i + x_{i-1}}{2}\right) \Delta x$$

$$= \frac{1}{2} \left(\frac{41}{16} + \frac{33}{16} + \frac{33}{16} + \frac{41}{16} \right)$$

$$= \frac{1}{2} \left(\frac{148}{16} \right) = \frac{74}{16} = \frac{37}{8}$$

Distance

$$- d = v \cdot t$$

$$= \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(t_i) \Delta t \right)$$

$$= \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(t_{i-1}) \Delta t \right)$$

$$= \lim_{n \rightarrow \infty} \left(\sum_{i=1}^n f(t_i^*) \Delta t \right)$$

