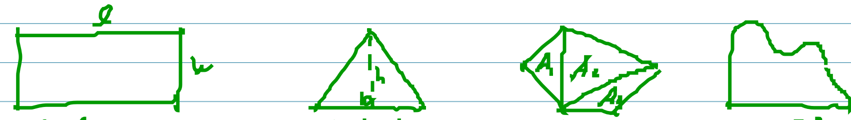
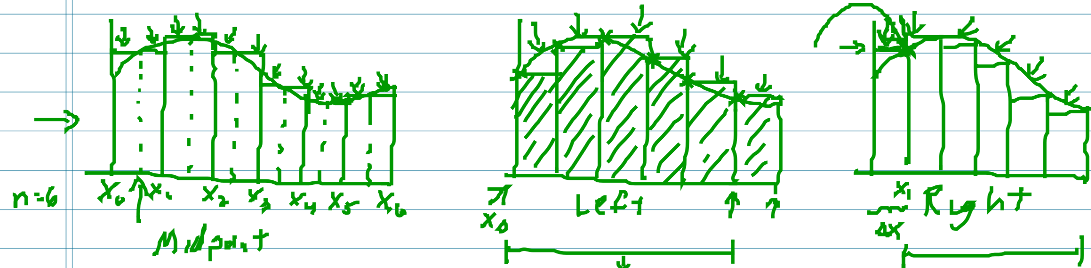
Area –



$A = lw$ $A = \frac{1}{2}bh$ $A = A_1 + A_2 + A_3$ $A = ??$



Sigma Notation

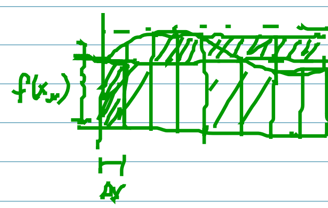
$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \dots + a_n$$

$$\sum_{i=3}^{5} (2i+1) = (2(3)+1) + (2(4)+1) + (2(5)+1)$$
$$= 7 + 9 + 11 = 27$$

$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x_n \qquad \Delta x = x_i - x_{i-1}$$
$$\Delta x_n = \frac{b-a}{n}$$

$$L_n = \sum_{i=1}^{n} f(x_{i-1})\,\Delta x_n$$

$$M_n = \sum_{i=1}^{n} f\left(\frac{x_i + x_{i-1}}{n}\right)\Delta x_n$$

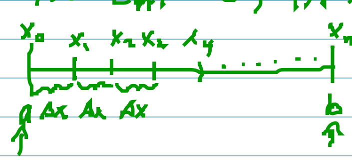


$$A = \lim_{n\to\infty}(R_n) = \lim_{n\to\infty}(L_n) = \lim_{n\to\infty}(M_n)$$

choose any small ε, can find N large enough st

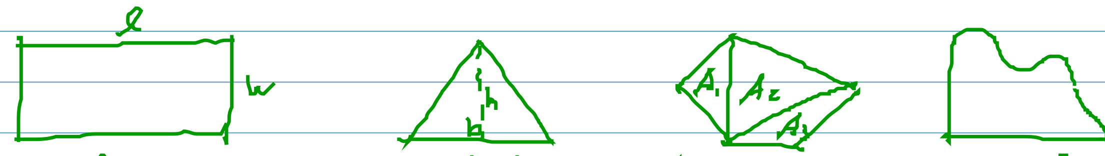
$\forall n \geq N$, $|A - R_n| < \varepsilon$, $|A - L_n| < \varepsilon$, $|A - M_n| < \varepsilon$

Sample point – x_i^*

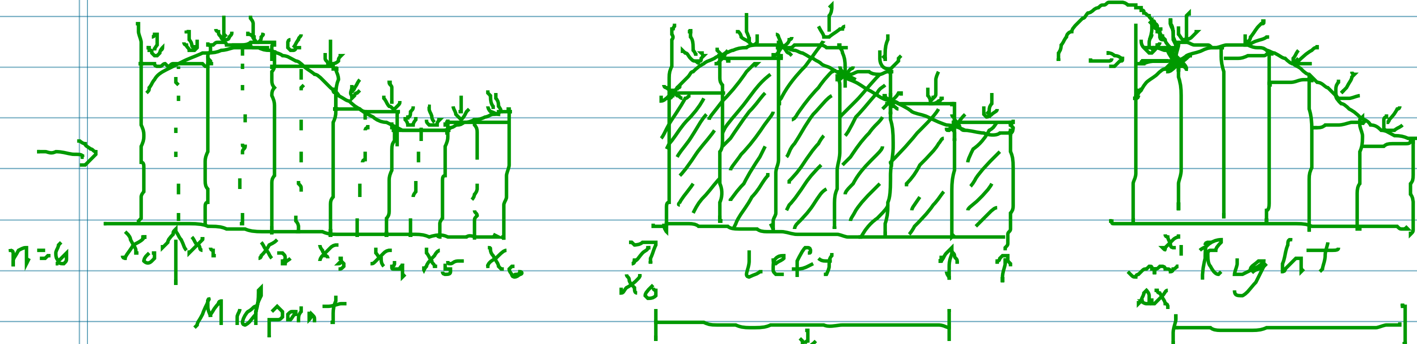


$x_0 = a$
$x_1 = a + \Delta x$
$x_2 = a + 2\Delta x$ → $x_i = a + i\Delta x$ $x_i^* \neq x_i$

Area –



$A = lw$ $\quad$ $A = \frac{1}{2}bh$ $\quad$ $A = A_1 + A_2 + A_3$ $\quad$ $A = ??$



Sigma Notation

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \dots + a_n$$

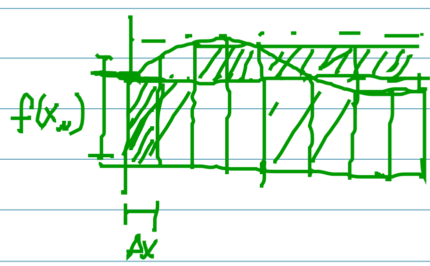
$$\sum_{i=3}^{5} (2i+1) = (2(3)+1) + (2(4)+1) + (2(5)+1)$$
$$= 7 + 9 + 11 = 27$$

$$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x_n \qquad \Delta x = x_i - x_{i-1}$$

$$\Delta x_n = \frac{b-a}{n}$$

$$L_n = \sum_{i=1}^{n} f(x_{i-1})\,\Delta x_n$$

$$M_n = \sum_{i=1}^{n} f\left(\frac{x_i + x_{i-1}}{n}\right)\Delta x_n$$

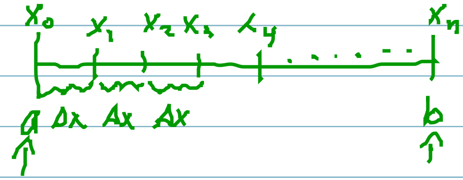


$$A = \lim_{n\to\infty}(R_n) = \lim_{n\to\infty}(L_n) = \lim_{n\to\infty}(M_n)$$

choose any small ε, can find N large enough st

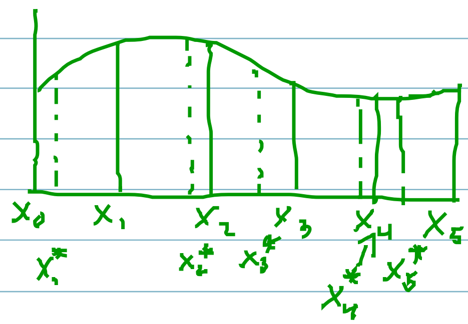
$\forall n \geq N$, $|A - R_n| < \varepsilon$, $|A - L_n| < \varepsilon$, $|A - M_n| < \varepsilon$

Sample point – x_i^*



$x_0 = a$
$x_1 = a + \Delta x$
$x_2 = a + 2\Delta x$ $\longrightarrow$ $x_i = a + i\Delta x$ $\qquad$ $x_i \neq x_i^*$

$$A = \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(x_i^*) \Delta x_i \right), \quad x_i^* \in (x_{i-1}, x_i)$$



R_4 $\quad f(x) = x^2 + 2 \quad [-1, 1]$ ($a = -1$, $b = 1$)

$$\Delta x_4 = \frac{1 - -1}{4} = \frac{2}{4} = \frac{1}{2}$$

$x_0 = -1$	$f(x_0) = 3$
$x_1 = -\frac{1}{2}$	$f(x_1) = \frac{9}{4}$
$x_2 = 0$	$f(x_2) = 2$
$x_3 = \frac{1}{2}$	$f(x_3) = \frac{9}{4}$
$x_4 = 1$	$f(x_4) = 3$

$$R_4 = \sum_{i=1}^{4} f(x_i) \Delta x$$
$$= f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$
$$= \frac{1}{2}\left(\frac{9}{4} + 2 + \frac{9}{4} + 3\right)$$
$$= \frac{1}{2}\left(5 + \frac{18}{4}\right) = \frac{1}{2}\left(\frac{38}{4}\right) = \frac{19}{4}$$

$$L_4 = \sum_{i=1}^{4} f(x_{i-1}) \Delta x = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x$$
$$= \frac{1}{2}\left(3 + \frac{9}{4} + 2 + \frac{9}{4}\right) = \frac{19}{4}$$

$x_1^* = -\frac{3}{4}$	$f(x_1^*) = \frac{41}{16}$
$x_2^* = -\frac{1}{4}$	$f(x_2^*) = \frac{33}{16}$
$x_3^* = \frac{1}{4}$	$f(x_3^*) = \frac{33}{16}$
$x_4^* = \frac{3}{4}$	$f(x_4^*) = \frac{41}{16}$

$$M_4 = \sum_{i=1}^{n} f\left(\frac{x_i + x_{i-1}}{2}\right) \Delta x$$
$$= \frac{1}{2}\left(\frac{41}{16} + \frac{33}{16} + \frac{33}{16} + \frac{41}{16}\right)$$
$$= \frac{1}{2}\left(\frac{148}{16}\right) = \frac{74}{16} = \frac{37}{8}$$

<u>Distance</u> $- \; d = vt$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(t_i) \Delta t \right)$$
$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(t_{i-1}) \Delta t \right)$$
$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(t_i^*) \Delta t \right)$$