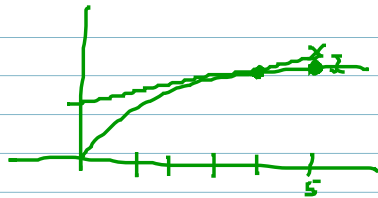


## Linear Approximation



Linearization -  $L(x) - y_1 = m(x - x_1)$   
 $L(x) - f(x_1) = f'(x_1)(x - x_1)$   
 $L(x) - f(a) = f'(a)(x - a)$   
 $L(x) = f'(a)(x - a) + f(a) = \underline{f'(a)x} + \underline{f(a) - f'(a)a}$

$f(x) = \sqrt{x}$       $f'(x) = \frac{1}{2}x^{-1/2}$       $\sqrt{5} = ?$   
 $a = 4$       $f'(4) = \frac{1}{2\sqrt{4}}$   
 $f(4) = 2$       $= \frac{1}{2 \cdot 2} = \frac{1}{4}$

$L(x) = \frac{1}{4}(x - 4) + 2 = \frac{1}{4}x - 1 + 2 = \frac{1}{4}x + 1$

$L(5) = \frac{1}{4}(5) + 1 = \frac{5}{4} + 1 = \frac{9}{4} = 2.25$       $\sqrt{5} \approx 2.236$

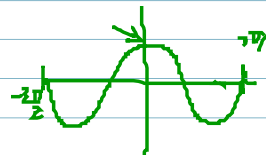
$\sin(x) \approx x$       $\cos(x) \approx 1$      Near 0

$f(x) = \sin(x)$	$g(x) = \cos(x)$
$f'(x) = \cos(x)$	$g'(x) = -\sin(x)$
$a = 0$	$a = 0$
$f(0) = 0$	$g(0) = 1$
$f'(0) = 1$	$g'(0) = 0$

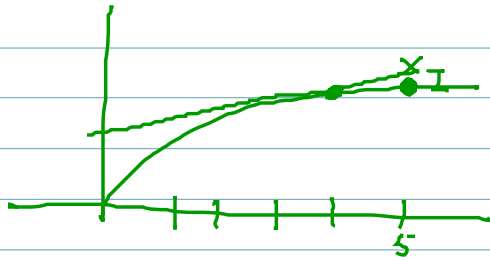
$L(x) = 1(x - 0) + 0$   
 $= x$



$L(x) = 0(x - 0) + 1$   
 $= 1$



# Linear Approximation



Linearization -  $L(x) - y_1 = m(x - x_1)$   
 $L(x) - f(x_1) = f'(x_1)(x - x_1)$   
 $L(x) - f(a) = f'(a)(x - a)$   
 $L(x) = f'(a)(x - a) + f(a) = \underline{f'(a)x} + \underline{f(a) - f'(a)a}$

$f(x) = \sqrt{x}$        $f'(x) = \frac{1}{2}x^{-1/2}$        $\sqrt{5} = ?$   
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$$L(x) = \frac{1}{4}(x - 4) + 2 = \frac{1}{4}x - 1 + 2 = \frac{1}{4}x + 1$$

$$L(5) = \frac{1}{4}(5) + 1 = \frac{5}{4} + 1 = \frac{9}{4} = 2.25 \quad \sqrt{5} \approx 2.236$$

$$\sin(x) \approx x$$

$$\cos(x) \approx 1$$

near 0

$$f(x) = \sin(x)$$

$$f'(x) = \cos(x)$$

$$a = 0$$

$$f(0) = 0$$

$$f'(0) = 1$$

$$g(x) = \cos(x)$$

$$g'(x) = -\sin(x)$$

$$a = 0$$

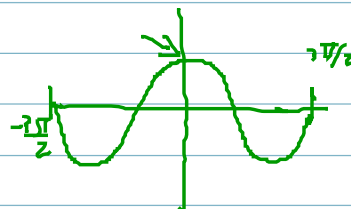
$$g(0) = 1$$

$$g'(0) = 0$$

$$L(x) = 1(x - 0) + 0$$
$$= x$$



$$L(x) = 0(x - 0) + 1$$
$$= 1$$



differential -  $dy, dx$

$$f'(x) = \frac{dy}{dx}$$

$\uparrow$   
 $\frac{d(y)}{dx}$

$$f(x) = y$$

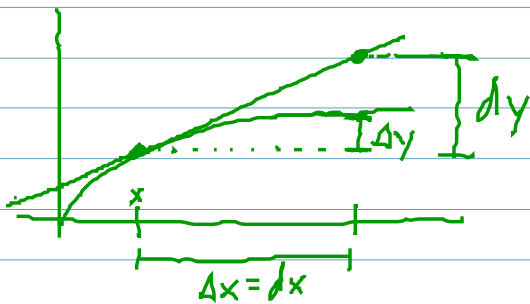
$\uparrow$

$$f'(x) dx = dy$$

$\uparrow$   
 $f'(x) x' = y'$

$$dx \neq \Delta x$$

$$dy \neq \Delta y$$



~~~~~

$$\underline{\text{Relative Error}} = \frac{\text{Actual} - \text{Approximate}}{\text{Actual}}$$

$$\frac{\Delta V}{V} \approx \frac{dV}{V} = \frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} = 3 \frac{dr}{r} \quad \text{Volume of sphere}$$

$$\underline{\text{Percentage Error}} = \text{Relative Error} \cdot 100$$

$$= \frac{\text{Actual} - \text{Approximate}}{\text{Actual}} \cdot 100$$