Chain Rule

If f, g = differentiable, $F(x) = (f \circ g)(x) = f(g(x))$,
Then F = differentiable w/ $F'(x) = f'(g(x))\, g'(x)$

$$\frac{d}{dx}(f(g(x))) = f'(g(x))\, g'(x)$$

$$\frac{d}{dx} f(u) = f'(u)\, u' \qquad u' = \frac{du}{dx}$$

$$\frac{d}{dx} f(x) = f'(x)\, x' = f'(x)(1) = f'(x)$$

$$\frac{dx}{dx} = 1 \qquad \frac{dx}{dt} \neq 1$$

Leibniz Notation – $y = f(u)$, $u = g(x)$

$$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx}$$

Power Rule w/ Chain Rule

If $n \in \mathbb{R}$, $u = g(x)$ = differentiable, Then $\frac{d}{dx}(u^n) = n u^{n-1} u'$ where $u' = \frac{du}{dx}$

$$\frac{d}{dx}(g(x))^n = n(g(x))^{n-1} g'(x)$$

$$f(x) = (5x+2)^3$$

$$f'(x) = 3(5x+2)^2 (5) = 15(5x+2)^2$$

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$\frac{d}{dx}$ ↑ $\qquad$ (1) ↑

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Leibniz Notation ~ $y = f(u)$, $u = g(x)$

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Power Rule w/ Chain Rule

If $n \in \mathbb{R}$, $u = g(x)$ = differentiable, Then $\frac{d}{dx}(u^n) = n u^{n-1} u'$

$u' = \frac{du}{dx}$

$$\frac{d}{dx}(g(x))^n = n(g(x))^{n-1} g'(x)$$

↑ $\qquad$ ↑

$f(x) = (5x+2)^3$

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↑ $\quad$ ↑ $\qquad$ ↓

$f(x) = \cos(x^2)$

$f'(x) = -\sin(x^2)\ 2x$

$f(x) = b^x = e^{\ln(b^x)} = e^{x\ln(b)}$

$f'(x) = e^{x\ln(b)}\ln(b) = b^x \ln(b)$

$f(x) = \sin^2(2x)$

$f'(x) = 2\sin(2x)\cos(2x)\,2 = 4\sin(2x)\cos(2x) = 2\sin(4x)$

$f(x) = \left(\frac{x^3-1}{x^3+1}\right)^5$

$f'(x) = 5\left(\frac{x^3-1}{x^3+1}\right)^4 \frac{(x^3+1)(3x^2) - (x^3-1)(3x^2)}{(x^3+1)^2}$

$f(x) = \sqrt{\frac{1+\cos(x)}{1+\sin(x)}} = \left(\frac{1+\cos(x)}{1+\sin(x)}\right)^{1/2}$

$f'(x) = \frac{1}{2}\left(\frac{1+\cos(x)}{1+\sin(x)}\right)^{-1/2} \frac{(1+\sin(x))(-\sin(x)) - (1+\cos(x))\cos(x)}{(1+\sin(x))^2}$

$f(x) = (5x-2)^7(3x+9)^8$

$f'(x) = (5x-2)^7\,8(3x+9)^7(3) + 7(5x-2)^6(5)(3x+9)^8$

$= 24(5x-2)^7(3x+9)^7 + 35(5x-2)^6(3x+9)^8$

$= (5x-2)^6(3x+9)^7[24(5x-2) + 35(3x+9)]$

$= (5x-2)^6(3x+9)^7[120x - 48 + 105x + 315] = (5x-2)^6(3x+9)^7(225x+267)$

$f(x) = e^{3x}\sin(5x)$

$f'(x) = e^{3x}\cos(5x)\,5 + \sin(5x)\,e^{3x}\,3$