Indefinite Integral

$\int f(x)dx = F(x)$ st $F'(x) = f(x)$

$\int f(x)dx = F(x) + c$

$\frac{d}{dx}(F(x)+c) = F'(x) + 0 = F'(x) = f(x)$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

$\int cf(x)dx = c\int f(x)dx$

$\int x^n dx = \frac{x^{n+1}}{x+1} + c$, $n \neq -1$

$\int \frac{1}{x}dx = \ln|x| + c$

$\int e^x dx = e^x + c$

$\int a^x dx = \frac{a^x}{\ln(a)} + c$

$\int \sin(x)dx = -\cos(x) + c$

$\int \cos(x)dx = \sin(x) + c$

$\int \sec^2(x)dx = \tan(x) + c$

$\int \csc^2(x)dx = -\cot(x) + c$

$\int \sec(x)\tan(x)dx = \sec(x) + c$

$\int \cot(x)\csc(x)dx = -\csc(x) + c$

$\int \frac{1}{x^2+1}dx = \tan^{-1}(x) + c$

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$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$$

$$\int \sinh(x)\,dx = \cosh(x) + C$$

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Net Change Theorem

The integral of a rate of change is the net change

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int_a^b v(t)\,dt = s(b) - s(a)$$

$$\int (3x^2 + 5x - 2)\,dx = \cancel{3}\frac{x^3}{\cancel{3}} + 5\frac{x^2}{2} - 2x + C$$

$$= x^3 + \tfrac{5}{2}x^2 - 2x + C$$

$$\int \frac{5x^7 + 3x^4 - 9x}{x^5}\,dx = \int\left(\frac{5x^7}{x^5} + \frac{3x^4}{x^5} - \frac{9x}{x^5}\right)dx = \int\left(5x^2 + \frac{3}{x} - \underbrace{\frac{9}{x^4}}_{-9x^{-4}}\right)dx$$

$$= 5\frac{x^3}{3} + 3\ln|x| - 9\frac{x^{-3}}{-3} + C = \tfrac{5}{3}x^3 + 3\ln|x| + \underbrace{3x^{-3}}_{\frac{3}{x^3}} + C$$

$$\int (x^2-3)(x+5)\,dx = \int (x^3 + 5x^2 - 3x - 15)\,dx$$

$$= \frac{x^4}{4} + 5\frac{x^3}{3} - 3\frac{x^2}{2} - 15x + C$$

$$\int \frac{\cos(\theta)}{\sin^2(\theta)}\,d\theta = \int \frac{1}{\sin(\theta)}\frac{\cos(\theta)}{\sin(\theta)}\,d\theta = \int \csc(\theta)\cot(\theta)\,d\theta$$

$$= -\csc(\theta) + C$$