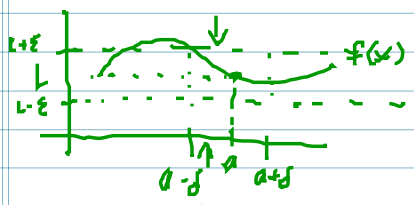
Precise Definition of a Limit

Let f be a function defined on some open interval that contains a, except possibly at a.

$\lim_{x \to a} (f(x)) = L$, if for every $\varepsilon > 0$, $\exists\ \delta > 0$ st if $0 < |x-a| < \delta$, Then $|f(x)-L| < \varepsilon$



Left-Hand Limit

$\lim_{x \to a^-} (f(x)) = L$ if $\forall \varepsilon > 0$, $\exists\ \delta > 0$ st
If $a - \delta < x < a$, Then $|f(x)-L| < \varepsilon$

Right-Hand Limit

$\lim_{x \to a^+} (f(x)) = L$ if $\forall \varepsilon > 0$, $\exists\ \delta > 0$ st
If $a < x < a+\delta$, Then $|f(x)-L| < \varepsilon$

Precise

Definition of Infinite Limits

Let f be a function defined on some open interval that contains a, except possibly at a,

$\lim_{x \to a} (f(x)) = \infty$ $\forall M > 0$, $\exists\ \delta > 0$ st
If $0 < |x-a| < \delta$, Then $f(x) > M$

$\lim_{x \to a} (f(x)) = -\infty$ $\forall N < 0$, $\exists\ \delta > 0$ st
If $0 < |x-a| < \delta$, Then $f(x) < N$

$\lim_{x \to 3} (4x-5) = 7$ $\quad 0 < |x-3| < \delta$ $\quad$ show $|(4x-5)-7| < \varepsilon$

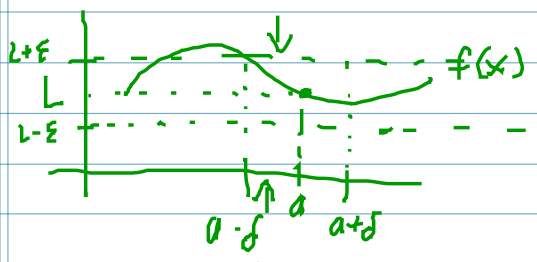
$|(4x-5)-7| = |4x-12| = |4(x-3)| = 4|x-3|$

$4|x-3| < \varepsilon$ $\quad |x-3| < \frac{\varepsilon}{4} = \delta$ ⇐

Precise Definition of a Limit

Let f be a function defined on some open interval that contains a, except possibly at a.

$\lim_{x \to a} (f(x)) = L$ if for every $\varepsilon > 0$, $\exists\ \delta > 0$ s.t. if $0 < |x-a| < \delta$, Then $|f(x)-L| < \varepsilon$



Left-Hand Limit

$\lim_{x \to a^-} (f(x)) = L$ if $\forall \varepsilon > 0$, $\exists\ \delta > 0$ s.t.
If $a - \delta < x < a$, Then $|f(x)-L| < \varepsilon$

Right-Hand Limit

$\lim_{x \to a^+} (f(x)) = L$ if $\forall \varepsilon > 0$, $\exists\ \delta > 0$ s.t.
If $a < x < a+\delta$, Then $|f(x)-L| < \varepsilon$

Precise Definition of Infinite Limit

Let f be a function defined on some open interval that contains a, except possibly at a.

$\lim_{x \to a} (f(x)) = \infty$ $\forall M > 0$, $\exists\ \delta > 0$ s.t.
If $0 < |x-a| < \delta$, Then $f(x) > M$

$\lim_{x \to a} (f(x)) = -\infty$ $\forall N < 0$, $\exists\ \delta > 0$ s.t.
If $0 < |x-a| < \delta$, Then $f(x) < N$

$\lim_{x \to 3} (4x-5) = 7$ $\quad 0 < |x-3| < \delta$ $\quad$ show $|(4x-5)-7| < \varepsilon$

$|(4x-5)-7| = |4x-12| \because |4(x-3)| = 4|x-3|$

$4|x-3) < \varepsilon$ $\quad |x-3) < \frac{\varepsilon}{4} = \delta \Leftarrow$