Extreme Values – Extrema

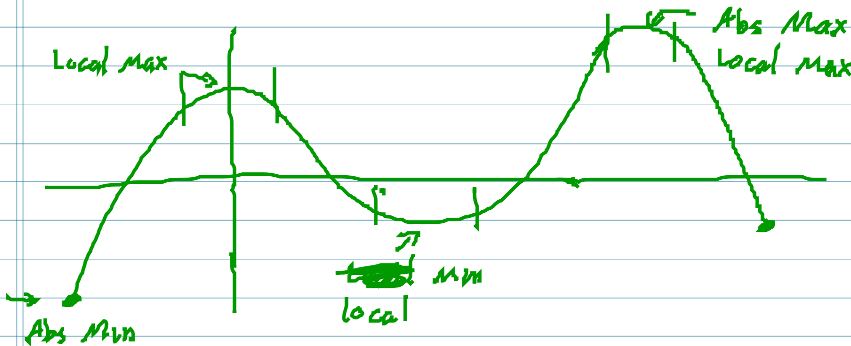
Absolute Maximum – at c st $f(c) \geq f(x)$, $\forall x \in D_f$

Absolute Minimum – at c st $f(c) \leq f(x)$, $\forall x \in D_f$

Local Maximum – at c st $f(c) \geq f(x)$, $\forall x \in (a,b)$ st $c \in (a,b)$ for some $(a,b) \subseteq D_f$

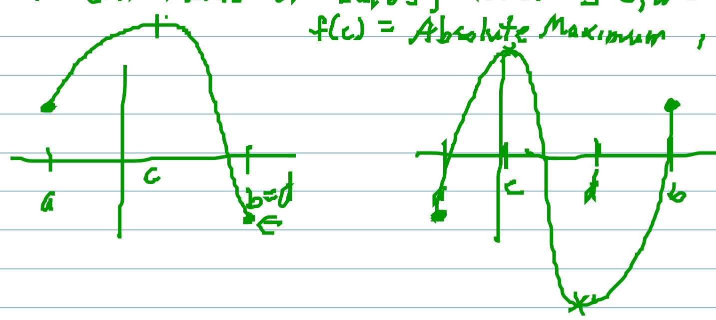
– "near c"

Local Minimum – at c st $f(c) \leq f(x)$, $\forall x \in (a,b)$ st $c \in (a,b)$ for some $(a,b) \subseteq D_f$



Extreme Value Theorem – EVT

If f = continuous on $[a,b]$, then $\exists\, c, d \in [a,b]$ st $f(c)$ = Absolute Maximum, $f(d)$ = Absolute Minimum



Fermat's Theorem

If $f(c)$ = Local Minimum/maximum, $f'(c)$ exists, Then $f'(c) = 0$

Extreme Values ~ Extrema

Absolute Maximum – at c st $f(c) \geq f(x)$, $\forall x \in D_f$

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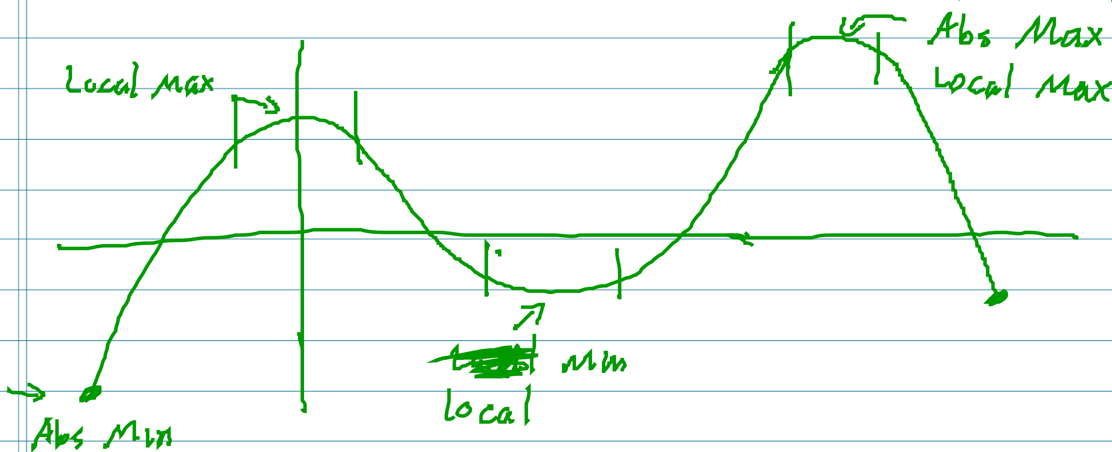
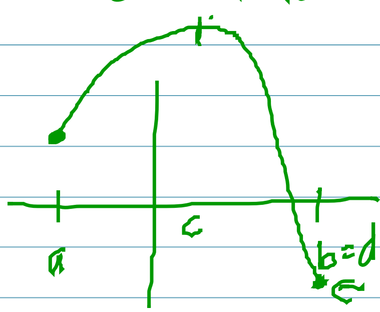
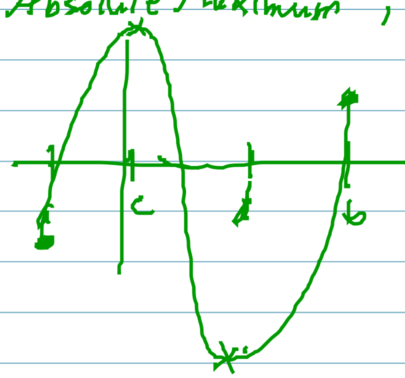
Local Minimum ~ at c st $f(c) \leq f(x)$, $\forall x \in (a,b)$ st $c \in (a,b)$ for some $(a,b) \subseteq D_f$



Extreme Value Theorem ~ EVT

If f = continuous on $[a,b]$, then $\exists\, c, d \in [a,b]$ st $f(c)$ = Absolute Maximum, $f(d)$ = Absolute Minimum





Fermat's Theorem

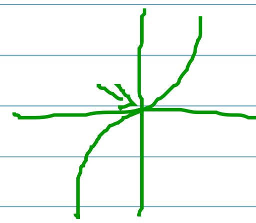
If $f(c)$ = Local Minimum/maximum, $f'(c)$ exists, Then $f'(c) = 0$

$f(x) = x^3$

$f'(x) = 3x^2$

$3x^2 = 0$

$x = 0$



Critical Number – $c \in D_f$ st $f'(c) = 0$ OR $f'(c)$ NOT exist

- If Local Minimum/Maximum at c, Then c = critical Number

Closed Interval Method

- Find $f'(x)$ & set $f'(x) = 0$ or $f'(x)$ does not exist
- Find value of f at critical numbers & endpoints
- Largest Value = Absolute Maximum
  Smallest Value = Absolute Minimum

$f(x) = 3x^4 - 16x^3 + 18x^2 \qquad [-2, 5]$

$f'(x) = 12x^3 - 48x^2 + 36x$

$= 12x(x^2 - 4x + 3)$

$= 12x(x-3)(x-1)$

$0 = 12x(x-3)(x-1) \qquad 12x = 0 \quad x - 3 = 0 \quad x - 1 = 0$

$x = 0 \quad x = 3 \quad x = 1$

$f(-2) = 3(-2)^4 - 16(-2)^3 + 18(-2)^2 = 48 + 128 + 72 = 248$

$f(0) = 0$

$f(1) = \cancel{12 - 48 + 36 = 0}\ 3 - 16 + 18 = 5$

$f(3) = 3(3)^4 - 16(3)^3 + 18(3)^2 = 243 - 432 + 162 = -27 \leftarrow$

$f(5) = 3(5)^4 - 16(5)^3 + 18(5)^2 = 1875 - 2000 + 450 = 325 \leftarrow$

27×16: 162, 27, 432

Absolute Max = $(5, 325)$

Absolute Min = $(3, -27)$

$f(x) = \dfrac{x-1}{x^2-2x+1} = \dfrac{x-1}{(x-1)^2} = \dfrac{1}{x-1}$

$\rightarrow x = 1 \leftarrow$ CN

$f'(x) = \dfrac{(x^2-2x+1)(1) - (x-1)(2x-2)}{(x^2-2x+1)^2} = \dfrac{x^2-2x+1 - (2x^2-4x+2)}{(x-1)^4}$

$= \dfrac{x^2-2x+1-2x^2+4x-2}{(x-1)^4} = \dfrac{-x^2+2x-1}{(x-1)^4} = -\dfrac{(x^2-2x+1)}{(x-1)^4} = -\dfrac{(x-1)^2}{(x-1)^4}$

$= -\dfrac{1}{(x-1)^2}$