Laws of Limits c = constant, $\lim_{x\to a}(f(x))$, $\lim_{x\to a}(g(x))$ Exist

Sum/Difference Law ~ $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a}(f(x)) \pm \lim_{x\to a}(g(x))$

Constant Multiple Law ~ $\lim_{x\to a}[c f(x)] = c \lim_{x\to a}(f(x))$

Product Law – $\lim_{x\to a}[f(x)g(x)] = [\lim_{x\to a}(f(x))][\lim_{x\to a}(g(x))]$

Quotient Law – $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a}(f(x))}{\lim_{x\to a}(g(x))}$, $\lim_{x\to a}(g(x)) \neq 0$

Power Law – $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a}(f(x))]^n$, $n \in \mathbb{N}$

$\lim_{x\to a}(c) = c$

$\lim_{x\to a}(x) = a$

$\lim_{x\to a}(x^n) = a^n$, $n \in \mathbb{N}$

$\lim_{x\to a}(\sqrt[n]{x}) = \sqrt[n]{a}$, $n \in \mathbb{N}$ If n: Even, then $a \geq 0$

Root Law $\lim_{x\to a}(\sqrt[n]{f(x)}) = \sqrt[n]{\lim_{x\to a}(f(x))}$, $n \in \mathbb{N}$ If n: Even, Then $\lim_{x\to a}(f(x)) > 0$

Direct Substitution Property
If $f(x)$ = Polynomial OR Rational, then $\lim_{x\to a}(f(x)) = f(a)$
Apply to trig functions
Apply for functions that are continuous at a

Theorem
If $f(x) = g(x)$, $\forall x \neq a$, Then $\lim_{x\to a}(f(x)) = \lim_{x\to a}(g(x))$

Theorem
$\lim_{x\to a}(f(x)) = L$ iff $\lim_{x\to a^-}(f(x)) = \lim_{x\to a^+}(f(x)) = L$

Laws of Limits

c = constant, $\lim_{x \to a}(f(x))$, $\lim_{x \to a}(g(x))$ Exist

Sum/Difference Law – $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a}(f(x)) \pm \lim_{x \to a}(g(x))$

Constant Multiple Law – $\lim_{x \to a}[cf(x)] = c\lim_{x \to a}(f(x))$

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Quotient Law – $\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a}(f(x))}{\lim_{x \to a}(g(x))}$, $\lim_{x \to a}(g(x)) \neq 0$

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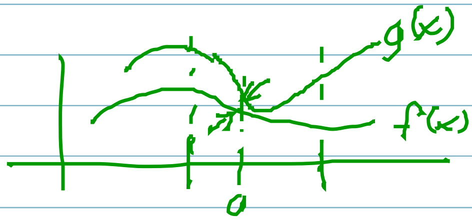
$\lim_{x \to a}(f(x)) = L$ iff $\lim_{x \to a^-}(f(x)) = \lim_{x \to a^+}(f(x)) = L$

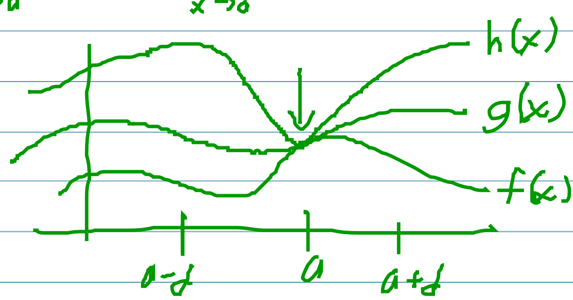
Theorem

If $f(x) \le g(x)$, $\forall x \in (a-\delta, a) \cup (a, a+\delta)$ for some $\delta > 0$, $\lim_{x\to a}(f(x))$, $\lim_{x\to a}(g(x))$ Exist, then $\lim_{x\to a}(f(x)) \le \lim_{x\to a}(g(x))$



Squeeze Theorem

If $f(x) \le g(x) \le h(x)$, $\forall x \in (a-\delta, a) \cup (a, a+\delta)$ for some $\delta > 0$, $\lim_{x\to a}(f(x)) = \lim_{x\to a}(h(x)) = L$, then $\lim_{x\to a}(g(x)) = L$



$\lim_{x\to 0}\left(x \sin\left(\frac{1}{x}\right)\right)$

$$-1 \le \sin\left(\tfrac{1}{x}\right) \le 1$$

$$\underbrace{-x}_{f(x)} \le \underbrace{x \sin\left(\tfrac{1}{x}\right)}_{g(x)} \le \underbrace{x}_{h(x)}$$

$\lim_{x\to 0}(-x) = 0$ $\qquad$ $\lim_{x\to 0}(x) = 0$ $\qquad$ $\lim_{x\to 0}\left(x \sin\left(\frac{1}{x}\right)\right) = 0$

$$\lim_{x\to 0}(-x) \le \lim_{x\to 0}\left(x \sin\left(\tfrac{1}{x}\right)\right) \le \lim_{x\to 0}(x)$$

$$0 \le \underbrace{\lim_{x\to 0}\left(x \sin\left(\tfrac{1}{x}\right)\right)}_{0} \le 0$$

Squeeze Theorem

$\lim_{x\to 0}\left(\frac{\sin(x)}{x}\right) = 1$