## Dot Product – scalar Product
Inner Product

$\vec{v} = \langle a_1, b_1 \rangle \quad \vec{w} = \langle a_2, b_2 \rangle$

$\vec{v} \cdot \vec{w} = a_1 a_2 + b_1 b_2$

**Properties**

$\vec{v} \cdot \vec{v} = |\vec{v}|^2$

$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$ — commutative

$\vec{u} \cdot (\vec{v} \cdot \vec{w}) = (\vec{u} \cdot \vec{v}) \cdot \vec{w}$ ← Associative

$\vec{u} \cdot (\vec{v} + \vec{w}) = (\vec{u} \cdot \vec{v}) + (\vec{v} \cdot \vec{w})$ – Distributive

$(c\vec{v}) \cdot \vec{w} = c(\vec{v} \cdot \vec{w}) = \vec{v} \cdot (c\vec{w})$

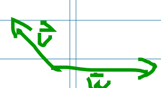
$\vec{0} \cdot \vec{v} = 0$

**Theorem**

If $\theta$ = angle between $\vec{v}$ + $\vec{w}$, Then $\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}|\cos(\theta)$

$\theta \in (0, \frac{\pi}{2})$, $\vec{v} \cdot \vec{w} > 0$

$\theta \in (\frac{\pi}{2}, \pi)$, $\vec{v} \cdot \vec{w} < 0$




**Corollary**

If $\theta$ = angle between $\vec{v}$ + $\vec{w}$, then $\cos(\theta) = \dfrac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|}$

**Parallel Vectors** – $\theta = 0$ or $\pi$

$\dfrac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|} = \pm 1 \qquad \vec{v} = c\vec{w}$

**Orthogonal** ~ $\theta = \frac{\pi}{2}$

$\dfrac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|} = 0$, so $\vec{v} \cdot \vec{w} = 0$

**Theorem**

$\vec{v} \cdot \vec{w} = 0$ iff $\vec{v}$ + $\vec{w}$ are orthogonal

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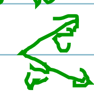

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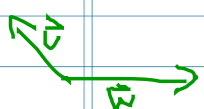

## Corollary

If $\theta$ = angle between $\vec{v}$ + $\vec{w}$, then $\cos(\theta) = \dfrac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|}$

## Parallel Vectors – $\theta = 0$ or $\pi$

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## Orthogonal – $\theta = \frac{\pi}{2}$

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## Theorem

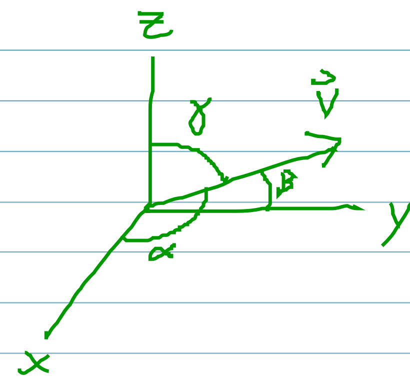
$\vec{v} \cdot \vec{w} = 0$ iff $\vec{v}$ + $\vec{w}$ are orthogonal

# Direction Angles

$\alpha, \beta, \gamma \in [0, \pi]$

$\uparrow \quad \uparrow \quad \uparrow$

$x \quad y \quad z$




$$\cos(\alpha) = \frac{\vec{v} \cdot \vec{i}}{|\vec{v}||\vec{i}|} = \frac{v_1}{|\vec{v}|}$$

$$\cos(\beta) = \frac{\vec{v} \cdot \vec{j}}{|\vec{v}||\vec{j}|} = \frac{v_2}{|\vec{v}|}$$

$$\cos(\gamma) = \frac{\vec{v} \cdot \vec{k}}{|\vec{v}||\vec{k}|} = \frac{v_3}{|\vec{v}|}$$

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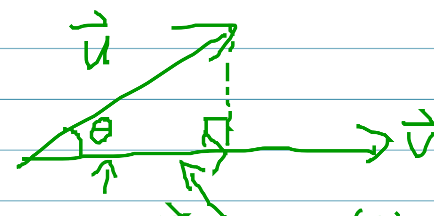
# Vector Projection of $\vec{u}$ onto $\vec{v}$

$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{v} \cdot \vec{u}}{|\vec{v}|} \frac{\vec{v}}{|\vec{v}|} = \frac{\vec{v} \cdot \vec{u}}{|\vec{v}|^2} \vec{v}$$

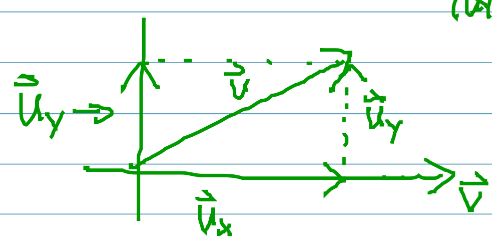



$\text{proj}_{\vec{v}} \vec{u}$ $\quad \cos(\theta) = \frac{x}{|\vec{u}|}$

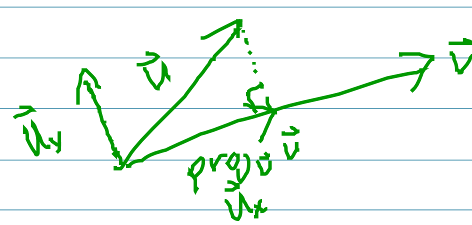
$$x = |\vec{u}| \cos(\theta)$$
$$= \cancel{|\vec{u}|} \frac{(\vec{u} \cdot \vec{v})}{\cancel{|\vec{u}|}|\vec{v}|}$$

# Decomposition of $\vec{u}$

$$\vec{u}_x = \left( \frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v}$$

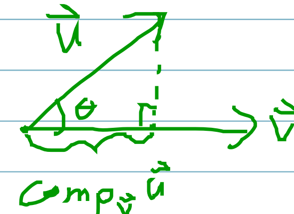



$$\vec{u}_x + \vec{u}_y = \vec{u} \longrightarrow \vec{u}_y = \vec{u} - \vec{u}_x$$



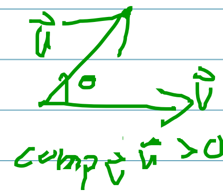
## Scalar Projection of $\vec{u}$ onto $\vec{v}$

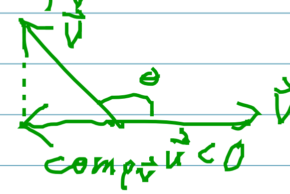
$$\text{comp}_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}|\vec{v}| = \boxed{\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}} \leftarrow$$



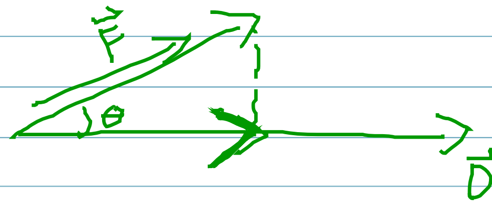

$$= \frac{\vec{u}\cdot\vec{v}}{|\vec{v}||\vec{u}|}|\vec{u}| = |\vec{u}|\cos(\theta)$$

$$\cos(\theta) = \frac{\text{comp}_{\vec{v}}\vec{u}}{|\vec{u}|}$$



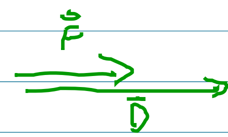




## Displacement Vector




## Work

$\vec{F} \parallel \vec{D}$




$$W = \vec{F}\cdot\vec{D} = |\vec{F}||\vec{D}|\cos(\theta) = |\vec{F}||\vec{D}| \qquad \theta = 0$$

$\vec{F} \nparallel \vec{D}$

$$W = (\text{Proj}_{\vec{D}}\vec{F})\cdot\vec{D} = (\text{comp}_{\vec{D}}\vec{F})|\vec{D}| = |\vec{F}|\cos(\theta)|\vec{D}| = |\vec{F}||\vec{D}|\cos(\theta)$$

---

$\vec{v} = \langle 3, 7, -2\rangle \qquad \vec{w} = \langle -1, 5, 6\rangle$

$\vec{v}\cdot\vec{w} = (3)(-1) + (7)(5) + (-2)(6) = -3 + 35 - 12 = 35 - 15 = 20$

$\vec{v} = \langle 5, 7, -1\rangle \qquad \vec{w} = \langle 2, k, 4\rangle$

$\vec{v}\cdot\vec{w} = 0 \qquad \vec{v}\cdot\vec{w} = 10 + 7k - 4 = 0$

$6 + 7k = 0 \rightarrow k = -\frac{6}{7}$