Symmetry

Even $\rightarrow$ $f(-x) = f(x)$

Odd $\rightarrow$ $f(x) = -f(-x)$ $\longrightarrow$ $f(-x) = -f(x)$

Asymptotes

Vertical Asymptotes $\lim_{x\to a^+} (f(x)) = \pm\infty$ $\lim_{x\to a^-} (f(x)) = \pm\infty$

$\rightarrow$ **Horizontal Asymptotes** $\lim_{x\to\infty} (f(x)) = L^+$ $\lim_{x\to-\infty} (f(x)) = L^-$

$\rightarrow$ **Oblique Asymptotes**

$f(x) = \frac{1}{x^2-4}$ $0 = \frac{1}{x^2-4}$ No x-int

$f(0) = -\frac{1}{4}$ y-int $(0, -\frac{1}{4})$

VA - $x^2-4=0$

$x = \pm 2$

$\uparrow$

$\lim_{x\to 2^+} \left(\frac{1}{x^2-4}\right) = \frac{1}{0^+} = \infty$

$\lim_{x\to 2^-} \left(\frac{1}{x^2-4}\right) = \frac{1}{0^-} = -\infty$

$\lim_{x\to -2^+} \left(\frac{1}{x^2-4}\right) = \frac{1}{0^-} = -\infty$

$\lim_{x\to -2^-} \left(\frac{1}{x^2-4}\right) = \frac{1}{0^+} = \infty$

HA - $\lim_{x\to\infty} \left(\frac{1}{x^2-4}\right) = 0$ $y=0$

$\lim_{x\to-\infty} \left(\frac{1}{x^2-4}\right) = 0$

No OA

$$f'(x) = \frac{(x^2-4)(0) - (1)(2x)}{(x^2-4)^2} = \frac{-2x}{(x^2-4)^2}$$

$\frac{-2x}{(x^2-4)^2} = 0$ $-2x = 0$ $x^2-4=0$

$\underline{x=0}$ $\underline{x = \pm 2}$

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Asymptotes

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$\rightarrow$ Horizontal Asymptotes $\lim_{x \to \infty} (f(x)) = L^+$ $\lim_{x \to -\infty} (f(x)) = L^-$

$\rightarrow$ Oblique Asymptotes

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$\lim_{x \to -2^-} \left(\frac{1}{x^2-4}\right) = \frac{1}{0^+} = \infty$

HA - $\lim_{y \to \infty} \left(\frac{1}{x^2-4}\right) = 0$ $y = 0$

$\lim_{x \to -\infty} \left(\frac{1}{x^2-4}\right) = 0$

No OA

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$\frac{-2x}{(x^2-4)^2} = 0$

$-2x = 0$

$x = 0$

$x^2-4=0$

$x = \pm 2$

$$f''(x) = \frac{(x^2-4)^{\cancel{2}}(-2) + (+2x)\,2\,\cancel{(x^2-4)}\,(2x)}{(x^2-4)^{\cancel{4}3}} = \frac{-2x^2+8+8x^2}{(x^2-4)^3} = \frac{6x^2+8}{(x^2-4)^3}$$

$6x^2+8=0$ $\qquad\qquad$ $(x^2-4)^3=0$

$x^2 = -\frac{4}{3}$ $\qquad\qquad$ $x = \pm 2$

No real answer

x		-2		0		2	
$f(x)$		X		$-\frac{1}{4}$		X	
$f'(x)$	+ ↗	X	↗ +	0 max	− ↘	X	− ↘
$f''(x)$	+ ∪	X		− ∩		X	+ ∪

$f'(-3) = \frac{-2(-3)}{((-3)^2-4)^2} = +$ $\qquad$ $f(1) = \frac{-2(1)}{(1^2-4)^2} = -$

$f'(-1) = \frac{-2(-1)}{[(-1)^2-4]^2} = +$ $\qquad$ $f(3) = \frac{-2(3)}{(3^2-4)^2} = -$

$f''(-3) = \frac{6(-3)^2+8}{[(-3)^2-4]^3} = +$ $\qquad$ $f''(0) = \frac{6(0)+8}{(0^2-4)^3} = -$ $\qquad$ $f(3) = \frac{6(3)^2+8}{(3^2-4)^3} = +$

Inc $-$ $(-\infty,-2) \cup (-2,0)$ $\qquad$ Local max $-$ $(0,-\frac{1}{4})$

Dec $-$ $(0,2) \cup (2,\infty)$

ccu $-$ $(-\infty,-2) \cup (2,\infty)$ $\qquad$ No IP

ccD $-$ $(-2,2)$