Derivative – $f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$

$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)$

– Differentiation operators – D, $\frac{d}{dx}$
↓
take derivative

Differentiable Function

– Differentiable at a – $f'(a)$ exist

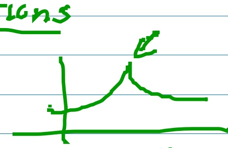
– Differentiable on interval – $f'(x)$ exist $\forall x \in I$ = interval

$I = (a,b), (-\infty, a), (a, \infty)$, OR $(-\infty, \infty)$

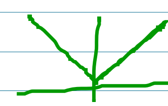
Theorem
If f = differentiable at a, Then f = continuous at a

Non-differentiable functions

– cusp/corner

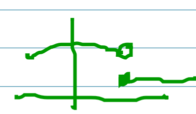


$\cos^2(x), \sin^2(x)$

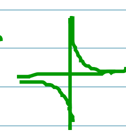


$|x|$

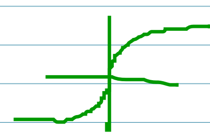
– Discontinuity



Piece-wise
$\frac{1}{x}$, $\frac{1}{x^2}$



– vertical Tangent



$\sqrt{x}$, $\sqrt[3]{x}$

Derivative

– $f'(x) = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)$$

– Differentiation operators – D, $\frac{d}{dx}$

↓

take derivative

Differentiable Function

– Differentiable at a – $f'(a)$ exist

– Differentiable on interval – $f'(x)$ exist $\forall x \in I$ = interval

$I = (a,b)$, $(-\infty, a)$, (a, ∞), or $(-\infty, \infty)$

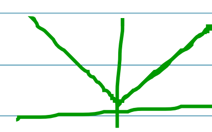
Theorem

If f = differentiable at a, then f = continuous at a

Non-differentiable functions

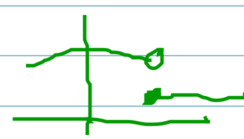
– cusp/corner

$\cos^2(x)$, $\sin^2(x)$



$|x|$

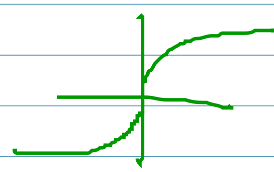
– Discontinuity



Piece-wise

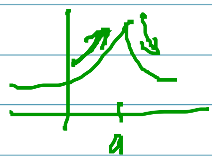
$\frac{1}{x}$, $\frac{1}{x^2}$

– vertical Tangent

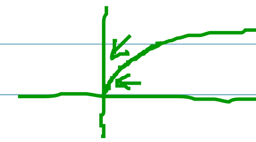


$\sqrt{x}$, $\sqrt[3]{x}$

- cusp/corner or discontinuity – $\lim_{x\to a^-}(f'(x)) \neq \lim_{x\to a^+}(f'(x))$




- vertical tangent – $\lim_{x\to a}|(f'(x))| = \infty$



Higher Derivatives

second derivative – $f''(x) = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$

$$= \lim_{h\to 0}\left(\frac{f'(x+h)-f'(x)}{h}\right)$$

- $f(x)$ – position
  $f'(x)$ – velocity $\quad v(x)$
  $f''(x)$ - acceleration $\quad v'(x) = a(x)$

Third derivative – $f'''(x) = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$

nth derivative – $f^{(n)}(x) = y^{(n)} = \frac{d^ny}{dx^n}$

$f(x) = \frac{1}{x-1}$

$$f'(x) = \lim_{h\to 0}\left(\frac{\frac{1}{x+h-1} - \frac{1}{x-1}}{h}\right) = \lim_{h\to 0}\left(\frac{x-1-(x+h-1)}{(x+h-1)(x-1)h}\right) = \lim_{h\to 0}\left(\frac{\cancel{x}\cancel{-1}\cancel{-x}-h\cancel{+1}}{(x+h-1)(x-1)h}\right)$$

$$= \lim_{h\to 0}\left(\frac{-\cancel{h}}{(x+h-1)(x-1)\cancel{h}}\right) = \lim_{h\to 0}\left(\frac{-1}{(x+h-1)(x-1)}\right) = \frac{-1}{(x-1)(x-1)}$$

$$= \frac{-1}{(x-1)^2}$$