

Composite function - $(f \circ g)(x) = f(g(x))$

Domain: $\{x \in D_g : g(x) \in D_f\}$

$$\begin{aligned} f(x) &= x^2 + 1 & f(g(x)) &= f(2x-3) = (2x-3)^2 + 1 \\ g(x) &= 2x - 3 & &= 4x^2 - 12x + 9 + 1 \\ & & &= 4x^2 - 12x + 10 \leftarrow \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(x^2+1) = 2(x^2+1) - 3 \\ &= 2x^2 + 2 - 3 \\ &= 2x^2 - 1 \leftarrow \end{aligned}$$

$$\begin{aligned} f(x) &= \sqrt{x-2} & f(g(x)) &= f(5x+1) = \sqrt{5x+1-2} = \sqrt{5x-1} \\ g(x) &= 5x+1 & & \uparrow \end{aligned}$$

$$\begin{aligned} 5x-1 &\geq 0 & x &\geq \frac{1}{5} \end{aligned}$$

$$\begin{aligned} x-2 &\geq 0 & x &\geq 2 \\ x &\geq 2 & \rightarrow g(x) &\geq 2 \rightarrow 5x+1 \geq 2 \\ & & & x \geq \frac{1}{5} \end{aligned}$$

$$g(f(x)) = g(\sqrt{x-2}) = 5\sqrt{x-2} + 1$$

$$x-2 \geq 0 \rightarrow x \geq 2$$

$$\begin{aligned} f(x) &= \sqrt{x+2} & f(g(x)) &= f(x^2+5) = \sqrt{x^2+5+2} = \sqrt{x^2+7} \\ g(x) &= x^2+5 & & \end{aligned}$$

$$\begin{aligned} x^2+7 &\geq 0 & x^2 &\geq -7 \leftarrow \end{aligned}$$

$$x+2 \geq 0 \rightarrow x \geq -2 \rightarrow g(x) \geq -2 \rightarrow x^2+5 \geq -2$$

$$g(f(x)) = g(\sqrt{x+2}) = (\sqrt{x+2})^2 + 5 = x+2+5 = x+7$$

$$\begin{aligned} &\uparrow \\ &x \geq -2 \end{aligned}$$

Composite function - $(f \circ g)(x) = f(g(x))$

Domain: $\{x \in D_g : g(x) \in D_f\}$

$$f(x) = x^2 + 1$$

$$g(x) = 2x - 3$$

$$\begin{aligned} f(g(x)) &= f(2x-3) = (2x-3)^2 + 1 \\ &= 4x^2 - 12x + 9 + 1 \\ &= 4x^2 - 12x + 10 \leftarrow \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(x^2+1) = 2(x^2+1) - 3 \\ &= 2x^2 + 2 - 3 \\ &= 2x^2 - 1 \leftarrow \end{aligned}$$

$$\begin{aligned} f(x) &= \sqrt{x-2} \\ g(x) &= 5x+1 \end{aligned}$$

$$f(g(x)) = f(5x+1) = \sqrt{5x+1-2} = \sqrt{5x-1}$$

$$5x-1 \geq 0$$

$$x \geq \frac{1}{5}$$

$$x-2 \geq 0$$

$$x \geq 2$$

$$x \geq 2 \rightarrow g(x) \geq 2 \rightarrow 5x+1 \geq 2$$

$$x \geq \frac{1}{5}$$

$$g(f(x)) = g(\sqrt{x-2}) = 5\sqrt{x-2} + 1$$

$$x-2 \geq 0 \rightarrow \underline{x \geq 2}$$

$$f(x) = \sqrt{x+2}$$

$$g(x) = x^2 + 5$$

$$f(g(x)) = f(x^2+5) = \sqrt{x^2+5+2} = \sqrt{x^2+7}$$

$$x^2+7 \geq 0$$

$$x^2 \geq -7 \leftarrow$$

$$x+2 \geq 0 \rightarrow x \geq -2 \rightarrow g(x) \geq -2 \rightarrow x^2+5 \geq -2$$

$$g(f(x)) = g(\sqrt{x+2}) = (\sqrt{x+2})^2 + 5 = x+2+5 = x+7$$

$$\uparrow$$
$$\textcircled{x \geq -2}$$

$$f(x) = x^2 + 1$$

$$g(x) = 2x - 3$$

$$f(g(5)) = f(7) = 50$$

$$f(g(5)) = 100 - 60 + 10 = 50$$

$$f(g(x)) = 4x^2 - 12x + 10 \leftarrow$$

$$g(f(-2)) = g(5) = 7$$

$$g(f(x)) = 2x^2 - 1$$

$$g(f(-2)) = 2(-2)^2 - 1 = 2(4) - 1 = 8 - 1 = 7$$

	↓		↓	↓	↓	↓	
X	-3	-2	-1	0	1	2	3
f(x)	0	3	-3	1	-2	-1	2
g(x)	3	-1	2	-2	-3	0	1

$$f(g(2)) = f(0) = 1$$

$$g(f(3)) = g(2) = 0$$

$$f(f(-3)) = f(0) = 1$$

$$g(g(-1)) = g(2) = 0$$

$$h(x) = f(g(x))$$

Fractions ←
 Parentheses ←
 Exponents
 Roots ←
 Multiplication ←

$$h(x) = \frac{1}{\sqrt{3x-5}}$$

$$f(x) = \frac{1}{x} \leftarrow$$

$$f(x) = \frac{1}{\sqrt{x}}$$

$$f(x) = \frac{1}{\sqrt{x-5}}$$

$$g(x) = \sqrt{3x-5}$$

$$g(x) = 3x-5$$

$$g(x) = 3x$$