$$\sin(2\theta) = \sin(\theta+\theta) = \sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta)$$
$$= 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos(\theta+\theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$$
$$\rightarrow = \cos^2(\theta) - \sin^2(\theta)$$
$$= 1-\sin^2(\theta) - \sin^2(\theta)$$
$$\rightarrow = 1 - 2\sin^2(\theta)$$
$$= \cos^2(\theta) + (-1+\cos^2(\theta))$$
$$\rightarrow = 2\cos^2(\theta) - 1$$

$$\tan(2\theta) = \tan(\theta+\theta) = \frac{\tan(\theta)+\tan(\theta)}{1-\tan(\theta)\tan(\theta)} = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

$\sin\left(\frac{u}{2}\right)$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ ($2\theta = u$, $\theta = \frac{u}{2}$)

$\sin(u) = 2\sin\left(\frac{u}{2}\right)\cos\left(\frac{u}{2}\right)$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$= 1 - 2\sin^2(\theta) \leftarrow$$
$$= 2\cos^2(\theta) - 1 \leftarrow$$

$\cos(u) = 1 - 2\sin^2\left(\frac{u}{2}\right)$

$\rightarrow \frac{1-\cos(u)}{2} = \sin^2\left(\frac{u}{2}\right) \leftarrow$

$\pm\sqrt{\frac{1-\cos(u)}{2}} = \sin\left(\frac{u}{2}\right)$

$\cos(u) = 2\cos^2\left(\frac{u}{2}\right) - 1$

$\frac{1+\cos(u)}{2} = \cos^2\left(\frac{u}{2}\right)$

$\pm\sqrt{\frac{1+\cos(u)}{2}} = \cos\left(\frac{u}{2}\right)$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)} = \frac{\pm\sqrt{\frac{1-\cos u}{2}}}{\pm\sqrt{\frac{1+\cos(u)}{2}}} = \pm\sqrt{\frac{1-\cos(u)}{1+\cos(u)}\cdot\frac{1-\cos(u)}{1-\cos(u)}} \leftarrow$$
$$= \pm\sqrt{\frac{(1-\cos(u))^2}{\sin^2(u)}} = \frac{1-\cos(u)}{\sin(u)} = \frac{\sin(u)}{1+\cos(u)}$$

$$\sin(2\theta) = \sin(\theta+\theta) = \sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta)$$
$$= 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos(\theta+\theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta)$$
$$\Rightarrow = \cos^2(\theta) - \sin^2(\theta)$$
$$= 1 - \sin^2(\theta) - \sin^2(\theta)$$
$$\Rightarrow = 1 - 2\sin^2(\theta)$$
$$= \cos^2(\theta) + (-1 + \cos^2(\theta))$$
$$\Rightarrow = 2\cos^2(\theta) - 1$$

$$\tan(2\theta) = \tan(\theta+\theta) = \frac{\tan(\theta)+\tan(\theta)}{1-\tan(\theta)\tan(\theta)} = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

$\sin\left(\frac{\theta}{2}\right)$

$\sin(\underbrace{2\theta}_{u}) = 2\sin(\underbrace{\theta}_{\frac{u}{2}})\cos(\underbrace{\theta}_{\frac{u}{2}})$

$\sin(u) = 2\sin\left(\frac{u}{2}\right)\cos\left(\frac{u}{2}\right)$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$
$$= 1 - 2\sin^2(\theta) \Leftarrow$$
$$= 2\cos^2(\theta) - 1 \Leftarrow$$

$\cos(u) = 1 - 2\sin^2\left(\frac{u}{2}\right)$

$\Rightarrow \frac{1-\cos(u)}{2} = \sin^2\left(\frac{u}{2}\right) \Leftarrow$

$\pm\sqrt{\frac{1-\cos(u)}{2}} = \sin\left(\frac{u}{2}\right)$

$\cos(u) = 2\cos^2\left(\frac{u}{2}\right) - 1$

$\frac{1+\cos(u)}{2} = \cos^2\left(\frac{u}{2}\right)$

$\pm\sqrt{\frac{1+\cos(u)}{2}} = \cos\left(\frac{u}{2}\right)$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)} = \frac{\pm\sqrt{\frac{1-\cos(u)}{2}}}{\pm\sqrt{\frac{1+\cos(u)}{2}}} = \pm\sqrt{\frac{1-\cos(u)}{1+\cos(u)}\cdot\frac{1-\cos(u)}{1-\cos(u)}} \Leftarrow$$

$$\therefore \pm\sqrt{\frac{(1-\cos(u))^2}{\sin^2(u)}} = \frac{1-\cos(u)}{\sin(u)} = \frac{\sin(u)}{1+\cos(u)}$$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

$= 1 - 2\sin^2(\theta)$

$= 2\cos^2(\theta) - 1$

$\tan(2\theta) = \dfrac{2\tan(\theta)}{1 - \tan^2(\theta)}$

$\sin^2(\theta) = \dfrac{1 - \cos(2\theta)}{2}$

$\cos^2(\theta) = \dfrac{1 + \cos(2\theta)}{2}$

$\tan^2(\theta) = \dfrac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - \cos(\theta)}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 + \cos(\theta)}{2}}$

$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - \cos(\theta)}{1 + \cos(\theta)}}$

$= \dfrac{1 - \cos(\theta)}{\sin(\theta)}$

$= \dfrac{\sin(\theta)}{1 + \cos(\theta)}$

$\sin\left(\frac{\pi}{12}\right) = \sqrt{\dfrac{1 - \cos\left(\frac{\pi}{6}\right)}{2}} = \sqrt{\dfrac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\dfrac{\frac{2 - \sqrt{3}}{2}}{2}} = \sqrt{\dfrac{2 - \sqrt{3}}{4}}$

$= \dfrac{\sqrt{2 - \sqrt{3}}}{2} = \dfrac{\sqrt{6} - \sqrt{2}}{4}$

$\cos\left(\frac{\pi}{8}\right) = \sqrt{\dfrac{1 + \cos\left(\frac{\pi}{4}\right)}{2}} = \sqrt{\dfrac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\dfrac{2 + \sqrt{2}}{4}} = \dfrac{\sqrt{2 + \sqrt{2}}}{2}$

$\tan\left(\frac{2\pi}{8}\right) = \dfrac{1 - \cos\left(\frac{2\pi}{4}\right)}{\sin\left(\frac{2\pi}{4}\right)} = \dfrac{1 - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \dfrac{\frac{2 - \sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \dfrac{2 - \sqrt{2}}{-\sqrt{2}} = \dfrac{2\sqrt{2} - 2}{-2}$

$= -\sqrt{2} + 1$

$= \dfrac{\sin\left(\frac{2\pi}{4}\right)}{1 + \cos\left(\frac{2\pi}{4}\right)} = \dfrac{-\frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} = \dfrac{-\frac{\sqrt{2}}{2}}{\frac{2 + \sqrt{2}}{2}} = \dfrac{-\sqrt{2}}{2 + \sqrt{2}} \cdot \dfrac{2 - \sqrt{2}}{2 - \sqrt{2}} = \dfrac{-2\sqrt{2} + 2}{4 - 2}$

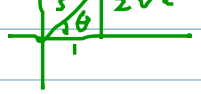
$= \dfrac{-2\sqrt{2} + 2}{2} = -\sqrt{2} + 1$

$\sec(\theta) = 3, \quad \sin(\theta) > 1$

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$= 2\left(\frac{2\sqrt{2}}{3}\right)\left(\frac{1}{3}\right)$

$= \dfrac{4\sqrt{2}}{9}$



$y^2 + 1^2 = 3^2$

$y^2 + 1 = 9$

$y^2 = 8$

$y = \pm 2\sqrt{2}$