

$$\begin{aligned}\sin(2\theta) &= \sin(\theta + \theta) = \sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta) \\ &= \underline{2\sin(\theta)\cos(\theta)}\end{aligned}$$

$$\begin{aligned}\cos(2\theta) &= \cos(\theta + \theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta) \\ &\rightarrow = \cos^2(\theta) - \sin^2(\theta) \\ &= 1 - \sin^2(\theta) - \sin^2(\theta) \\ &\rightarrow = 1 - 2\sin^2(\theta) \\ &= \cos^2(\theta) + (-1 + \cos^2(\theta)) \\ &\rightarrow = 2\cos^2(\theta) - 1\end{aligned}$$

$$\tan(2\theta) = \tan(\theta + \theta) = \frac{\tan(\theta) + \tan(\theta)}{1 - \tan(\theta)\tan(\theta)} = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\begin{aligned}\sin\left(\frac{\theta}{2}\right) \\ \sin(2\theta) &= 2\sin(\theta)\cos(\theta) \\ \sin(u) &= 2\sin\left(\frac{u}{2}\right)\cos\left(\frac{u}{2}\right)\end{aligned}$$

$$\begin{aligned}\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= 1 - 2\sin^2(\theta) \leftarrow \\ &= 2\cos^2(\theta) - 1 \leftarrow\end{aligned}$$

$$\begin{aligned}\cos(u) &= 1 - 2\sin^2\left(\frac{u}{2}\right) & \cos(u) &= 2\cos^2\left(\frac{u}{2}\right) - 1 \\ \rightarrow \frac{1 - \cos(u)}{2} &= \sin^2\left(\frac{u}{2}\right) \leftarrow & \frac{1 + \cos(u)}{2} &= \cos^2\left(\frac{u}{2}\right) \\ \pm \sqrt{\frac{1 - \cos(u)}{2}} &= \sin\left(\frac{u}{2}\right) \uparrow & \pm \sqrt{\frac{1 + \cos(u)}{2}} &= \cos\left(\frac{u}{2}\right)\end{aligned}$$

$$\begin{aligned}\tan\left(\frac{u}{2}\right) &= \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)} = \frac{\pm \sqrt{\frac{1 - \cos(u)}{2}}}{\pm \sqrt{\frac{1 + \cos(u)}{2}}} = \pm \sqrt{\frac{1 - \cos(u)}{1 + \cos(u)}} \leftarrow \\ &= \pm \sqrt{\frac{(1 - \cos(u))^2}{\sin^2(u)}} = \frac{1 - \cos(u)}{\sin(u)} = \frac{\sin(u)}{1 + \cos(u)}\end{aligned}$$

$$\underline{\sin(2\theta)} = \sin(\theta + \theta) = \sin(\theta)\cos(\theta) + \cos(\theta)\sin(\theta) \\ = \underline{2\sin(\theta)\cos(\theta)}$$

$$\cos(2\theta) = \cos(\theta + \theta) = \cos(\theta)\cos(\theta) - \sin(\theta)\sin(\theta) \\ \rightarrow = \cos^2(\theta) - \sin^2(\theta) \\ = 1 - \sin^2(\theta) - \sin^2(\theta) \\ \rightarrow = 1 - 2\sin^2(\theta) \\ = \cos^2(\theta) + (-1 + \cos^2(\theta)) \\ \rightarrow = 2\cos^2(\theta) - 1$$

$$\tan(2\theta) = \tan(\theta + \theta) = \frac{\tan(\theta) + \tan(\theta)}{1 - \tan(\theta)\tan(\theta)} = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin\left(\frac{\theta}{2}\right)$$

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\sin(u) = 2 \sin\left(\frac{u}{2}\right) \cos\left(\frac{u}{2}\right)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \\ = 1 - 2\sin^2(\theta) \leftarrow \\ = 2\cos^2(\theta) - 1 \leftarrow$$

$$\rightarrow \frac{1 - \cos(u)}{2} = \sin^2\left(\frac{u}{2}\right) \leftarrow$$

$$\pm \sqrt{\frac{1 - \cos(u)}{2}} = \sin\left(\frac{u}{2}\right)$$

$$\cos(u) = 2\cos^2\left(\frac{u}{2}\right) - 1 \\ \} + \cos(u) = \cos^2\left(\frac{u}{2}\right)$$

$$\pm \sqrt{\frac{1 + \cos(u)}{2}} = \cos\left(\frac{u}{2}\right)$$

$$\tan\left(\frac{u}{2}\right) = \frac{\sin\left(\frac{u}{2}\right)}{\cos\left(\frac{u}{2}\right)} = \frac{\pm \sqrt{\frac{1 - \cos(u)}{2}}}{\pm \sqrt{\frac{1 + \cos(u)}{2}}} = \pm \sqrt{\frac{1 - \cos(u)}{1 + \cos(u)}} \leftarrow$$

$$= \pm \sqrt{\frac{(1 - \cos(u))^2}{\sin^2(u)}} = \frac{1 - \cos(u)}{\sin(u)} = \frac{\sin(u)}{1 + \cos(u)}$$

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$= 1 - 2\sin^2(\theta)$$

$$= 2\cos^2(\theta) - 1$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1 - \tan^2(\theta)}$$

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2(\theta) = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos(\theta)}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{1 + \cos(\theta)}}$$

$$= \frac{1 - \cos(\theta)}{\sin(\theta)}$$

$$= \frac{\sin(\theta)}{1 + \cos(\theta)}$$

$$\sin\left(\frac{\pi}{12}\right) = \sqrt{\frac{1 - \cos\left(\frac{\pi}{6}\right)}{2}} = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{2}} = \sqrt{\frac{2 - \sqrt{3}}{4}}$$

$$= \frac{\sqrt{2 - \sqrt{3}}}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\cos\left(\frac{\pi}{8}\right) = \sqrt{\frac{1 + \cos\left(\frac{\pi}{4}\right)}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{\sqrt{2 + \sqrt{2}}}{2}$$

$$\tan\left(\frac{7\pi}{8}\right) = \frac{1 - \cos\left(\frac{7\pi}{4}\right)}{\sin\left(\frac{7\pi}{4}\right)} = \frac{1 - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \frac{2 - \sqrt{2}}{-\sqrt{2}} = \frac{2 - \sqrt{2}}{-\sqrt{2}} = \frac{2\sqrt{2} - 2}{-2}$$

$$= -\sqrt{2} + 1$$

$$= \frac{\sin\left(\frac{7\pi}{4}\right)}{1 + \cos\left(\frac{7\pi}{4}\right)} = \frac{-\frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}} = \frac{-\frac{\sqrt{2}}{2}}{\frac{2 + \sqrt{2}}{2}} = \frac{-\sqrt{2}}{2 + \sqrt{2}} \cdot \frac{2 - \sqrt{2}}{2 - \sqrt{2}} = \frac{-2\sqrt{2} + 2}{4 - 2}$$

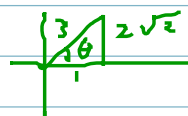
$$= \frac{-2\sqrt{2} + 2}{2} = -\sqrt{2} + 1$$

$$\sec(\theta) = 3, \sin(\theta) > 1$$

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$= 2 \left(\frac{2\sqrt{2}}{3}\right) \left(\frac{1}{3}\right)$$

$$= \frac{4\sqrt{2}}{9}$$



$$y^2 + 1^2 = 3^2$$

$$y^2 + 1 = 9$$

$$y^2 = 8$$

$$y = \pm 2\sqrt{2}$$