$f(x) = \sin(x)$

$f'(x) = \lim_{h\to 0}\left(\frac{\sin(x+h) - \sin(x)}{h}\right) = \lim_{h\to 0}\left[\frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}\right]$

$= \lim_{h\to 0}\left(\frac{\sin(x)(\cos(h)-1) + \cos(x)\sin(h)}{h}\right)$

$= \sin(x)\lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) + \cos(x)\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$

$= \sin(x)(0) + \cos(x)(1) = \cos(x)$

$g(x) = \cos(x)$

$g'(x) = \lim_{h\to 0}\left(\frac{\cos(x+h) - \cos(x)}{h}\right) = \lim_{h\to 0}\left(\frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}\right)$

$= \lim_{h\to 0}\left(\frac{\cos(x)(\cos(h)-1) - \sin(x)\sin(h)}{h}\right)$

$= \cos(x)\lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) - \sin(x)\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$

$= \cos(x)(0) - \sin(x)(1) = -\sin(x)$

$f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$

$f'(x) = \frac{\cos(x)(\cos(x)) - \sin(x)(-\sin(x))}{(\cos(x))^2} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$

$= \frac{1}{\cos^2(x)} = \sec^2(x)$

$f(x) = \csc(x) = \frac{1}{\sin(x)}$

$f'(x) = \frac{\sin(x)(0) - (1)\cos(x)}{[\sin(x)]^2} = -\frac{\cos(x)}{\sin^2(x)} = -\frac{1}{\sin(x)}\frac{\cos(x)}{\sin(x)} = -\csc(x)\cot(x)$

$f(x) = \sin(x)$

$$f'(x) = \lim_{h\to 0}\left(\frac{\sin(x+h) - \sin(x)}{h}\right) = \lim_{h\to 0}\left(\frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sin(x)(\cos(h)-1) + \cos(x)\sin(h)}{h}\right)$$

$$= \sin(x)\lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) + \cos(x)\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$$

$$= \sin(x)(0) + \cos(x)(1) = \cos(x)$$

$g(x) = \cos(x)$

$$g'(x) = \lim_{h\to 0}\left(\frac{\cos(x+h) - \cos(x)}{h}\right) = \lim_{h\to 0}\left(\frac{\cos(x)\cos(h) - \sin(x)\sin(h) - \cos(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\cos(x)(\cos(h)-1) - \sin(x)\sin(h)}{h}\right)$$

$$= \cos(x)\lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right) - \sin(x)\lim_{h\to 0}\left(\frac{\sin(h)}{h}\right)$$

$$= \cos(x)(0) - \sin(x)(1) = -\sin(x)$$

$f(x) = \tan(x) = \dfrac{\sin(x)}{\cos(x)}$

$$f'(x) = \frac{\cos(x)(\cos(x)) - \sin(x)(-\sin(x))}{(\cos(x))^2} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos^2(x)} = \sec^2(x)$$

$f(x) = \csc(x) = \dfrac{1}{\sin(x)}$

$$f'(x) = \frac{\sin(x)(0) - (1)\cos(x)}{(\sin(x))^2} = -\frac{\cos(x)}{\sin^2(x)} = -\frac{1}{\sin(x)}\,\frac{\cos(x)}{\sin(x)} = -\csc(x)\cot(x)$$

$$f(x) = \sec(x) = \frac{1}{\cos(x)}$$

$$f'(x) = \frac{\cos(x)\,(0) - (1)\,(-\sin(x))}{(\cos(x))^2} = \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos(x)}\,\frac{\sin(x)}{\cos(x)}$$

$$= \sec(x)\tan(x)$$

$$f(x) = \cot(x) = \frac{\cos(x)}{\sin(x)}$$

$$f'(x) = \frac{\sin(x)(-\sin(x)) - \cos(x)\cos(x)}{(\sin(x))^2} = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)}$$

$$= -\frac{(\sin^2(x) + \cos^2(x))}{\sin^2(x)} = \frac{-1}{\sin^2(x)} = -\csc^2(x)$$

$$\frac{d}{dx}(\sin(x)) = \cos(x) \qquad \frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x) \qquad \frac{d}{dx}(\cot(x)) = -\csc^2(x)$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x) \qquad \frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$$

$$\lim_{x \to 0}\left(\frac{\sin(5x)}{3x}\right) = \lim_{x \to 0}\left(\frac{5\sin(5x)}{3(x\,5)}\right) = \frac{5}{3}\underbrace{\lim_{x \to 0}\left(\frac{\sin(5x)}{5x}\right)} = \frac{5}{3}$$

$$\lim_{x \to 0}(x\cot(x)) = \lim_{x \to 0}\left(x\,\frac{\cos(x)}{\sin(x)}\right) = \lim_{x \to 0}\left(\cos(x)\,\frac{x}{\sin(x)}\right) = 1 \cdot 1 = 1$$

$0 \to$ $f(x) = \sin(x)$

$f'(x) = \cos(x)$

$f''(x) = -\sin(x)$

$f'''(x) = -\cos(x)$

$4 \to$ $f^{(4)}(x) = \sin(x)$

$$f^{(511)}(x) = f'''(x) = -\cos(x)$$

$$\frac{511}{4} = 127\tfrac{3}{4} \Rightarrow 127 \text{ R } 3$$