Law of Natural Growth/Decay

$-\frac{dy}{dt} = Ky$, K = constant

$K > 0$ Grow
$K < 0$ Decay

$y = Ce^{kt}$, $C = y(0)$ = initial condition

Theorem

The ONLY solution to $\frac{dy}{dt} = ky$ is $y(t) = y(0)e^{kt}$

$y_0 \leftarrow$ y naught

Population growth

$P(t) = P_0 e^{kt}$

Relative growth Rate – $\frac{1}{P}\frac{dP}{dt} = K$

$\frac{dP}{dt} = KP$

Radioactive Decay

$\rightarrow m(t) = m_0 e^{kt}$, $K < 0$

Decay Rate $= -\frac{1}{m}\frac{dm}{dt} = K$, m = mass

Half-Life – $\frac{1}{2}m_0 = m_0 e^{kt}$

$\frac{1}{2} = e^{kt}$

Newton's Law of Cooling

$T(t) = T_s + (T_0 - T_s)e^{kt}$, $K < 0$

$T(t)$ = Temperature at t
T_s = surrounding temp
T_0 = original temp

$T(t) - T_s = (T_0 - T_s)e^{kt}$

$y(t) = y_0 e^{kt}$

$\frac{dT}{dt} = k(T - T_s)$

$\lim_{t\to\infty} (T(t)) = T_s$

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$\frac{dT}{dt} = K(T(t) - T_s)$

$\lim_{t \to \infty}(T(t)) = T_s$

Compound Interest

$I = Prt$

P = Principle
r = rate
t = time

$A = P + I$
$= P + Pr$
$= P(1+r)$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

n = number of times compounded

$$\lim_{n\to\infty}\left(P\left(1+\frac{r}{n}\right)^{nt}\right) = Pe^{rt}$$

$$\lim_{n\to\infty}\left(\left(1+\frac{r}{n}\right)^{n}\right) = \lim_{n\to\infty}\left(e^{\ln\left(\left(1+\frac{r}{n}\right)^{n}\right)}\right) = e^{r}$$

$$\lim_{n\to\infty}\left(\ln\left(\left(1+\frac{r}{n}\right)^{n}\right)\right)$$

$$\lim_{n\to\infty}\left(n\ln\left(1+\frac{r}{n}\right)\right)$$

$$\lim_{n\to\infty}\left(\frac{\ln\left(1+\frac{r}{n}\right)}{\frac{1}{n}}\right) \overset{H}{=} \lim_{n\to\infty}\left(\frac{\frac{1}{1+\frac{r}{n}}\left(+\frac{r}{\cancel{n^2}}\right)}{+\frac{1}{\cancel{n^2}}}\right) = \lim_{n\to\infty}\left(\frac{r}{1+\frac{r}{n}}\right) = r$$

$A = Pe^{rt}$ continuous interest

$$\lim_{n\to\infty}\left(\left(1+\frac{r}{n}\right)^{n}\right) = e^{r} \qquad \lim_{n\to\infty}\left(\left(1+\frac{1}{n}\right)^{n}\right) = e^{1}$$

$N(t) = N_0 e^{Kt}$

$K>0$ grow
$K<0$ Decay

$$\frac{dN}{dt} = KN(t)$$