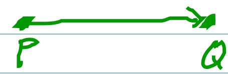
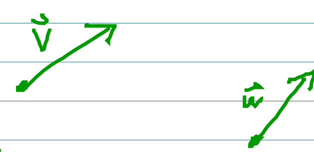
Vector – quantity w/ magnitude (length) + direction

Geometric – $\vec{PQ} = \vec{PQ}$

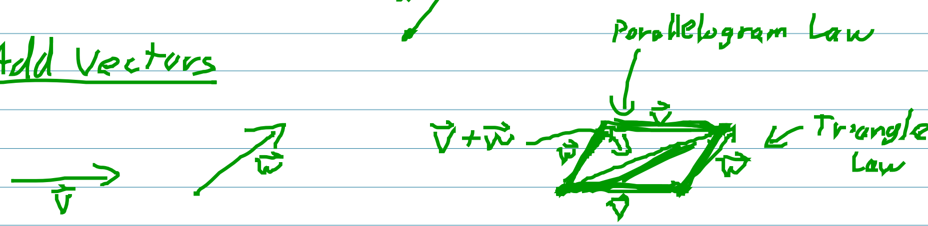


Zero vector – $\vec{0} = 0$

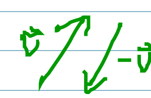
Equal – $\vec{v} = \vec{w}$ iff Magnitude + direction are same for $\vec{v}$ + $\vec{w}$



Add Vectors

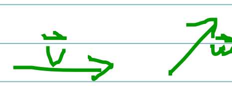


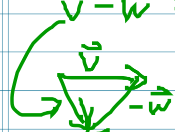
commutative – $\vec{v}+\vec{w} = \vec{w}+\vec{v}$
associative – $\vec{u}+(\vec{v}+\vec{w}) = (\vec{u}+\vec{v})+\vec{w}$
identity – $\vec{v}+\vec{0} = \vec{0}+\vec{v} = \vec{v}$
inverse – $\vec{v}+(-\vec{v}) = (-\vec{v})+\vec{v} = \vec{0}$



Subtract Vectors

$\vec{v}-\vec{w} = \vec{v}+(-\vec{w})$



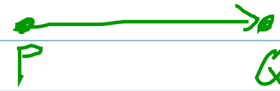


Scalar ~ Real number

Scalar Product ~ $\alpha\vec{v}$

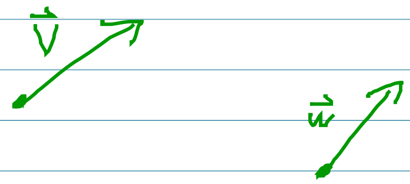
Vector – quantity w/ magnitude + direction
"length"

Geometric – $\overrightarrow{PQ} = \overrightarrow{PQ}$

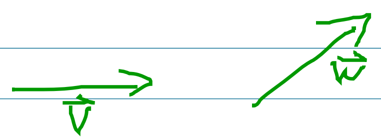


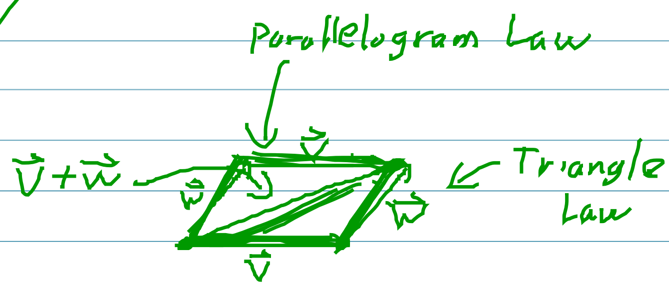
Zero vector – $\vec{0} = 0$

Equal – $\vec{v} = \vec{w}$ iff Magnitude + direction are same for $\vec{v}$ + $\vec{w}$

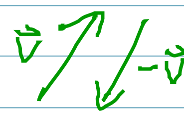


Add Vectors



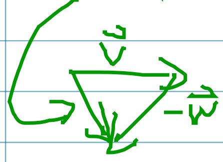


commutative – $\vec{v} + \vec{w} = \vec{w} + \vec{v}$
associative – $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$
identity – $\vec{v} + \vec{0} = \vec{0} + \vec{v} = \vec{v}$
inverse – $\vec{v} + (-\vec{v}) = (-\vec{v}) + \vec{v} = \vec{0}$



subtract Vectors

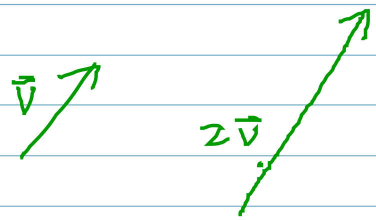
$\vec{v} - \vec{w} = \vec{v} + (-\vec{w})$



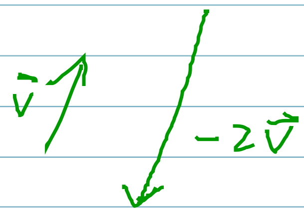
Scalar – Real number

scalar Product – $\alpha \vec{v}$

$\alpha > 0$, $\alpha\vec{v}$ has magnitude α times ~~longer than~~ the magnitude of $\vec{v}$
same direction as $\vec{v}$



$\alpha < 0$, $\alpha\vec{v}$ has magnitude α times ~~longer than~~ the magnitude of $\vec{v}$
opposite direction as $\vec{v}$



$\alpha = 0$, $\alpha\vec{v} = \vec{0}$

$(0)\vec{v} = \vec{0}$ $\quad (1)\vec{v} = \vec{v}$ $\quad (-1)\vec{v} = -\vec{v}$

$(\alpha + \beta)\vec{v} = \alpha\vec{v} + \beta\vec{v}$
$\alpha(\vec{v} + \vec{w}) = \alpha\vec{v} + \alpha\vec{w}$
$\alpha(\beta\vec{v}) = (\alpha\beta)\vec{v}$

Magnitude $-$ $\|\vec{v}\| = |\vec{v}|$

$|\vec{v}| \geq 0$ $\qquad |-\vec{v}| = |\vec{v}|$
$|\vec{v}| = 0$ iff $\vec{v} = \vec{0}$ $\qquad |\alpha\vec{v}| = |\alpha|\,|\vec{v}|$

Unit Vector $-$ $\vec{u}$ st $|\vec{u}| = 1$

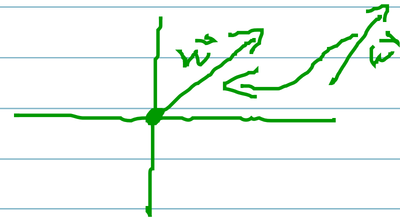
Parallel vectors $-$ scalar multiples of each other

Algebraic Vector $-$ $\vec{v} = \langle a, b \rangle$, $a, b \in \mathbb{R}$
components

$\vec{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$
$P = (x_1, y_1)$ $\quad Q = (x_2, y_2)$

Position Vector

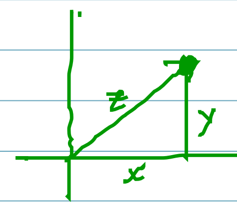
$\langle \underline{x_2 - x_1}, \underline{y_2 - y_1} \rangle$



Equality – If $\vec{v} = \langle a_1, b_1 \rangle$ & $\vec{w} = \langle a_2, b_2 \rangle$, then $\vec{v} = \vec{w}$ iff $a_1 = a_2$ & $b_1 = b_2$

Magnitude – $|\vec{v}| = \sqrt{a^2 + b^2}$ 2-D

$= \sqrt{a^2 + b^2 + c^2}$ 3-D



Add/subtract

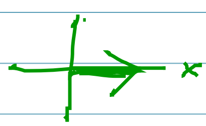
$\vec{v} = \langle a_1, b_1 \rangle \qquad \vec{w} = \langle a_2, b_2 \rangle$

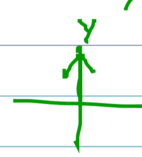
$\vec{v} \pm \vec{w} = \langle a_1 \pm a_2, b_1 \pm b_2 \rangle$

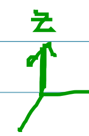
$c\vec{v} = \langle ca_1, cb_1 \rangle$

Direction Vectors – standard basis Vectors

$\vec{i} = \langle 1, 0, 0 \rangle \qquad \vec{j} = \langle 0, 1, 0 \rangle \qquad \vec{k} = \langle 0, 0, 1 \rangle$







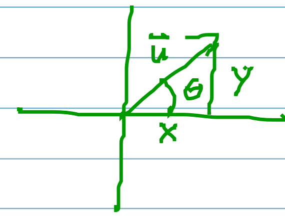
$\vec{v} = \langle a, b, c \rangle = a\vec{i} + b\vec{j} + c\vec{k}$

$\vec{v} \pm \vec{w} = \langle a_1, b_1 \rangle \pm \langle a_2, b_2 \rangle = \langle a_1 \pm a_2, b_1 \pm b_2 \rangle$

$= (a_1 \pm a_2)\vec{i} + (b_1 \pm b_2)\vec{j}$

$\vec{u} = \dfrac{\vec{v}}{|\vec{v}|} \Rightarrow \vec{v} = |\vec{v}|\,\vec{u}$

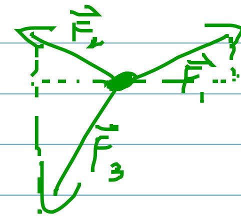
$\vec{u} = \langle \cos(\theta), \sin(\theta) \rangle$
$= \cos(\theta)\vec{i} + \sin(\theta)\vec{j}$



$\vec{v} = |\vec{v}| \left(\cos(\theta)\vec{i} + \sin(\theta)\vec{j}\right)$

Resultant Force

$\vec{F}_r = \sum_{i=1}^{n} \vec{F}_i$



Static Equilibrium - object at Rest

$\vec{F}_r = \vec{0}$

Dot Product - Scalar Product ←
Inner Product

$\vec{v} = \langle a_1, b_1 \rangle$ $\quad\quad$ $\vec{w} = \langle a_2, b_2 \rangle$

$\vec{v} \cdot \vec{w} = a_1 a_2 + b_1 b_2$

~~Pro~~ **Properties**

$\vec{v} \cdot \vec{v} = |\vec{v}|^2$
$\vec{v} \cdot \vec{w} = \vec{w} \cdot \vec{v}$ — commutative
$\vec{u} \cdot (\vec{v} \cdot \vec{w}) = (\vec{u} \cdot \vec{v}) \cdot \vec{w}$ — Associative
$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$ — Distributive
$(c\vec{v}) \cdot \vec{w} = c(\vec{v} \cdot \vec{w}) = \vec{v} \cdot (c\vec{w})$
$\vec{0} \cdot \vec{v} = 0$

Theorem

If θ = angle between $\vec{v}$ + $\vec{w}$, Then $\vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}| \cos(\theta)$

$\theta \in (0, \frac{\pi}{2})$, $\vec{v} \cdot \vec{w} > 0$
$\theta \in (\frac{\pi}{2}, \pi)$, $\vec{v} \cdot \vec{w} < 0$