Continuous at a

$\lim_{x \to a} (f(x)) = f(a)$

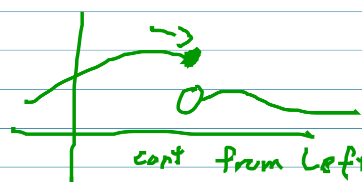
- $f(a)$ exists, $a \in D_f$
- $\lim_{x \to a} (f(x))$ exists

Discontinuous at a

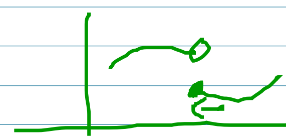
- Removable – can remove the discontinuity by redefining $f(a)$
  - $\lim_{x \to a} (f(x)) \neq f(a)$
- Infinite – $\lim_{x \to a^+} (f(x)) = \pm\infty$ And/or $\lim_{x \to a^-} (f(x)) = \pm\infty$
- Jump – $\lim_{x \to a^+} (f(x)) \neq \lim_{x \to a^-} (f(x))$, so $\lim_{x \to a} (f(x))$ does not exist

Continuous from Left – $\lim_{x \to a^-} (f(x)) = f(a)$

Continuous from Right – $\lim_{x \to a^+} (f(x)) = f(a)$



cont from Left



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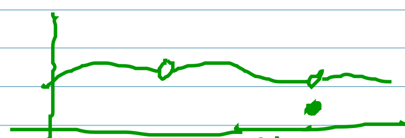


Removable

Continuous on an Interval

continuous $\forall x \in I$ = interval

Continuous at a

$\lim_{x \to a} (f(x)) = f(a)$

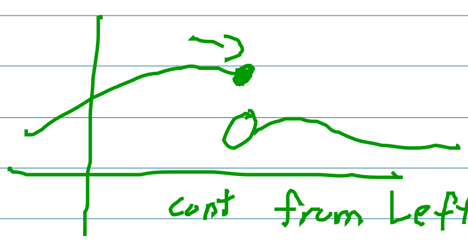
- $f(a)$ exists , $a \in D_f$
- $\lim_{x \to a} (f(x))$ exists

Discontinuous at a

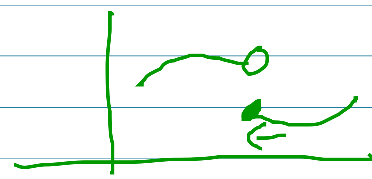
- Removable - can remove the discontinuity by redefining $f(a)$
  - $\lim_{x \to a} (f(x)) \neq f(a)$
- Infinite - $\lim_{x \to a^+} (f(x)) = \pm\infty$ And/or $\lim_{x \to a^-} (f(x)) = \pm\infty$
- Jump - $\lim_{x \to a^+} (f(x)) \neq \lim_{x \to a^-} (f(x))$, so $\lim_{x \to a} (f(x))$ does not exist

Continuous from Left - $\lim_{x \to a^-} (f(x)) = f(a)$

continuous from Right - $\lim_{x \to a^+} (f(x)) = f(a)$



cont from Left



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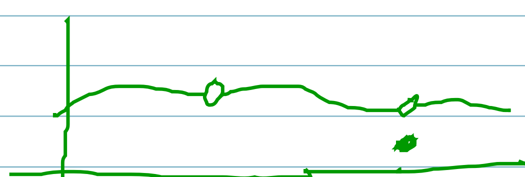


Removable

Continuous on an Interval

continuous $\forall x \in I$ = interval

Theorem

If f, g = continuous at a & c = constant,
Then $f \pm g$, cf, fg, $\frac{f}{g}$ are all continuous at a
$g(a) \neq 0$

Theorem

Polynomial = continuous everywhere
Rational Function = continuous on domain
Root Functions, Trig Functions = continuous on domain

Theorem

If f = continuous at b, $\lim_{x \to a}(g(x)) = b$, Then $\lim_{x \to a}(f(g(x)) = f(b)$

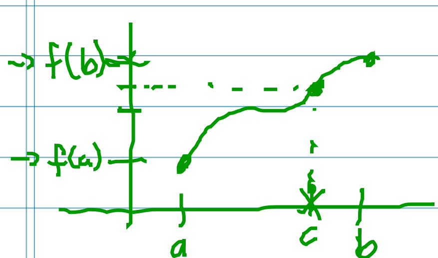
$$\lim_{x \to a}(f(g(x)) = f(\lim_{x \to a}(g(x)))$$

Theorem

If g = continuous at a, f = continuous at $g(a)$, then
$f \circ g$ = continuous at a $\quad (f \circ g)(x) = f(g(x))$

Intermediate Value Theorem

If f = continuous on $[a, b]$, $f(a) \neq f(b)$, $N \in (f(a), f(b))$
OR $N \in (f(b), f(a))$, Then $\exists\, c \in (a, b)$ st $f(c) = N$



$f(a) > 0$, $f(b) < 0$
$f(a) < 0$, $f(b) > 0$