

Quotient

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\sec(\theta)}{\csc(\theta)} \quad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{\csc(\theta)}{\sec(\theta)}$$

Reciprocal

$$\begin{aligned} \sin(\theta) &= \frac{1}{\csc(\theta)} & \csc(\theta) &= \frac{1}{\sin(\theta)} & \tan(\theta) &= \frac{1}{\cot(\theta)} \\ \cos(\theta) &= \frac{1}{\sec(\theta)} & \sec(\theta) &= \frac{1}{\cos(\theta)} & \cot(\theta) &= \frac{1}{\tan(\theta)} \end{aligned}$$

Pythagorean

$$\sin^2(\theta) + \cos^2(\theta) = 1 \quad \tan^2(\theta) + 1 = \sec^2(\theta) \quad 1 + \cot^2(\theta) = \csc^2(\theta)$$

Odd/Even

$$\begin{aligned} \sin(-\theta) &= -\sin(\theta) & \cos(-\theta) &= \cos(\theta) & \tan(-\theta) &= -\tan(\theta) \\ \csc(-\theta) &= -\csc(\theta) & \sec(-\theta) &= \sec(\theta) & \cot(-\theta) &= -\cot(\theta) \end{aligned}$$

Cofunction

$$\begin{aligned} \sin\left(\frac{\pi}{2} - \theta\right) &= \cos(\theta) & \cos\left(\frac{\pi}{2} - \theta\right) &= \sin(\theta) & \tan\left(\frac{\pi}{2} - \theta\right) &= \cot(\theta) \\ \csc\left(\frac{\pi}{2} - \theta\right) &= \sec(\theta) & \sec\left(\frac{\pi}{2} - \theta\right) &= \csc(\theta) & \cot\left(\frac{\pi}{2} - \theta\right) &= \tan(\theta) \end{aligned}$$

$$\begin{aligned} \frac{\cos(\theta)}{1 + \sin(\theta)} &= \frac{1 - \sin(\theta)}{\cos(\theta)} \\ &= \frac{1 - \sin(\theta)}{\cos(\theta)} \cdot \frac{1 + \sin(\theta)}{1 + \sin(\theta)} && \text{multiply by "1"} \\ &= \frac{1 - \sin^2(\theta)}{\cos(\theta)(1 + \sin(\theta))} && \text{FOIL} \\ &= \frac{\cos^2(\theta)}{2\cos(\theta)(1 + \sin(\theta))} && \text{Pythagorean ID} \\ &= \frac{\cos(\theta)}{1 + \sin(\theta)} && \text{cancel} \end{aligned}$$

$$\csc(\theta) \cdot \tan(\theta) = \sec(\theta)$$

$$\begin{aligned} \frac{1}{\sin(\theta)} \cdot \frac{\sin(\theta)}{\cos(\theta)} &= && \text{Reciprocal ID, Quotient ID} \\ \frac{1}{\cancel{\sin(\theta)}} \cdot \frac{\cancel{\sin(\theta)}}{\cos(\theta)} &= && \text{cancel} \\ \sec(\theta) &= && \text{Reciprocal ID} \end{aligned}$$

Quotient

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\sec(\theta)}{\csc(\theta)}$$

$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{\csc(\theta)}{\sec(\theta)}$$

Reciprocal

$$\sin(\theta) = \frac{1}{\csc(\theta)}$$

$$\cos(\theta) = \frac{1}{\sec(\theta)}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)}$$

$$\tan(\theta) = \frac{1}{\cot(\theta)}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)}$$

Pythagorean

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Odd/Even

$$\sin(-\theta) = -\sin(\theta)$$

$$\csc(-\theta) = -\csc(\theta)$$

$$\cos(-\theta) = \cos(\theta)$$

$$\sec(-\theta) = \sec(\theta)$$

$$\tan(-\theta) = -\tan(\theta)$$

$$\cot(-\theta) = -\cot(\theta)$$

Cofunction

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta)$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc(\theta)$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta)$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$$

$$\frac{\cos(\theta)}{1 + \sin(\theta)} = \frac{1 - \sin(\theta)}{\cos(\theta)}$$

$$= \frac{1 - \sin(\theta)}{\cos(\theta)} \cdot \frac{1 + \sin(\theta)}{1 + \sin(\theta)}$$

multiply by "1"

$$= \frac{1 - \sin^2(\theta)}{\cos(\theta)(1 + \sin(\theta))}$$

FOIL

$$= \frac{\cos^2(\theta)}{2\cos(\theta)(1 + \sin(\theta))}$$

Pythagorean ID

$$= \frac{\cos(\theta)}{1 + \sin(\theta)}$$

cancel

$$\csc(\theta) \cdot \tan(\theta) = \sec(\theta)$$

$$\frac{1}{\sin(\theta)} \cdot \frac{\sin(\theta)}{\cos(\theta)} =$$

$$\frac{1}{\cos(\theta)} =$$

$$\sec(\theta) =$$

Reciprocal ID, Quotient ID

cancel

Reciprocal ID

$$\frac{1}{1-\cos(\theta)} + \frac{1}{1+\cos(\theta)} = 2 \csc^2(\theta)$$

$$\frac{1+\cos(\theta)}{1+\cos(\theta)} \cdot \frac{1}{1-\cos(\theta)} + \frac{1}{1+\cos(\theta)} \cdot \frac{1-\cos(\theta)}{1-\cos(\theta)} =$$

Common Denominator

$$\frac{1+\cos(\theta) + 1-\cos(\theta)}{1-\cos^2(\theta)} =$$

FOIL

$$\frac{2}{1-\cos^2(\theta)} =$$

simplify

$$\frac{2}{\sin^2(\theta)} =$$

Pythagorean ID

$$2 \csc^2(\theta) =$$

Reciprocal ID

$$\sin(\theta) (\cot(\theta) + \tan(\theta)) = \sec(\theta)$$

$$\sin(\theta) \left(\frac{\cos(\theta)}{\sin(\theta)} + \frac{\sin(\theta)}{\cos(\theta)} \right) =$$

Quotient ID

$$\sin(\theta) \left(\frac{\cos^2(\theta) + \sin^2(\theta)}{\sin(\theta)\cos(\theta)} \right) =$$

Common Denominators

$$\sin(\theta) \left(\frac{1}{\sin(\theta)\cos(\theta)} \right) =$$

Pythagorean ID

$$\frac{1}{\cos(\theta)} =$$

cancel

$$\sec(\theta) =$$

Reciprocal ID

$$\sin\left(\theta - \frac{\pi}{2}\right) = -\cos(-\theta)$$

$$\sin\left(-\left(\frac{\pi}{2} - \theta\right)\right) =$$

Factor

$$-\sin\left(\frac{\pi}{2} - \theta\right) =$$

Odd/Even

$$-\cos(\theta) =$$

cofunction

$$-\cos(-\theta) =$$

odd/Even

$$\frac{\sin(\theta)}{\sin(\theta) - \cos(\theta)} = \frac{1}{1 - \cot(\theta)}$$

cancel

$$\frac{1 - \cot(\theta)}{1 - \cot(\theta)} \cdot \frac{\sin(\theta)}{\sin(\theta) - \cos(\theta)} =$$

Multiply by "1"

$$\frac{\sin(\theta) - \frac{\cos(\theta)}{\sin(\theta)} \sin(\theta)}{(1 - \cot(\theta))(\sin(\theta) - \cos(\theta))} =$$

Distribute, Quotient ID

$$\frac{\sin(\theta) - \cos(\theta)}{(1 - \cot(\theta))(\sin(\theta) - \cos(\theta))} =$$

cancel