Velocity

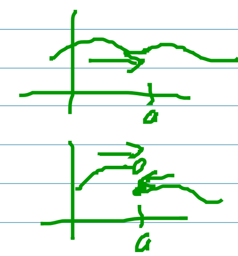
Average Velocity — $\dfrac{\text{change in position}}{\text{change in time}}$

Instantaneous Velocity — no change in time

limiT ~ $\lim_{x \to a} (f(x)) = L$

- As x gets closer & closer to a (from both sides of a), $f(x)$ gets closer to L
- $\lim_{x \to a} (f(x)) \neq f(a)$

Sided Limits



Left-Hand Limit — $\lim_{x \to a^-} (f(x)) = L$

- $x < a$

Right-Hand Limit - $\lim_{x \to a^+} (f(x)) = L$

- $x > a$

$\lim_{x \to a} (f(x)) = L$ iff $\lim_{x \to a^-} (f(x)) = \lim_{x \to a^+} (f(x)) = L$

- If $\lim_{x \to a^-} (f(x)) \neq \lim_{x \to a^+} (f(x))$, Then $\lim_{x \to a} (f(x))$ Does Not Exist

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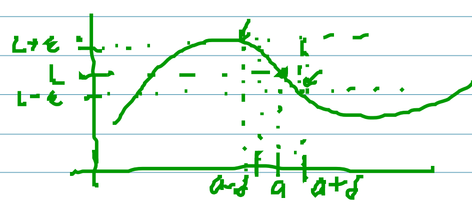
ε-δ Definition

ε - epsilon $\quad \delta$ - delta

Given $\varepsilon > 0$, $\exists\, \delta > 0$ st if $|x-a| < \delta$, then $|f(x) - L| < \varepsilon$

$\lim_{x \to a} (f(x)) = L$



Velocity

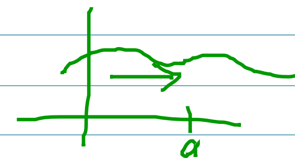
Average Velocity — $\dfrac{\text{change in position}}{\text{change in time}}$

Instantaneous Velocity — No change in time

Limit — $\lim_{x \to a} (f(x)) = L$

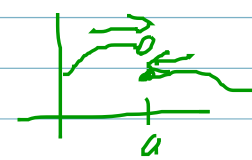
- As x gets closer & closer to a (from both sides of a), f(x) gets closer to L
- $\lim_{x \to a} (f(x)) \neq f(a)$

Sided Limits



Left-Hand Limit — $\lim_{x \to a^-} (f(x)) = L$

— $x < a$



Right-Hand Limit — $\lim_{x \to a^+} (f(x)) = L$

— $x > a$

$\lim_{x \to a} (f(x)) = L$ iff $\lim_{x \to a^-} (f(x)) = \lim_{x \to a^+} (f(x)) = L$

- If $\lim_{x \to a^-} (f(x)) \neq \lim_{x \to a^+} (f(x))$, Then $\lim_{x \to a} (f(x))$ Does Not Exist

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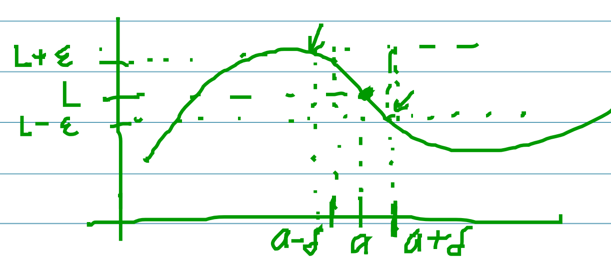
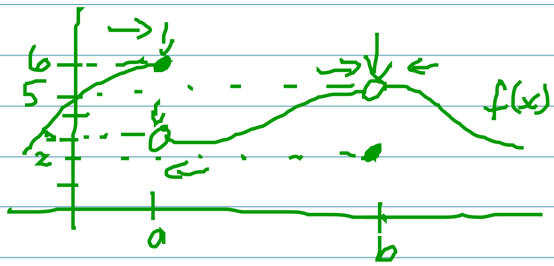
ε-δ Definition

ε - epsilon δ - delta

Given $\varepsilon > 0$, $\exists\, \delta > 0$ st if $|x-a| < \delta$, then $|f(x) - L| < \varepsilon$

$\lim_{x \to a} (f(x)) = L$





$\lim_{x\to a^-}(f(x)) = 6$

$\lim_{x\to a^+}(f(x)) = 3$

$\lim_{x\to a}(f(x)) = DNE$

$\lim_{x\to b^-}(f(x)) = 5$ $\quad \lim_{x\to b^+}(f(x)) = 5$ $\quad \lim_{x\to b}(f(x)) = 5$ $\quad f(b) = 2$

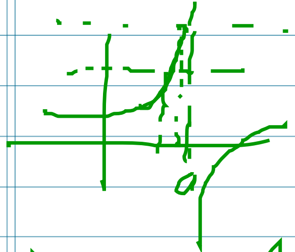
$\cancel{\lim_{x\to 2}\left(\frac{x^2-4}{\text{[illegible]}}\right)}$ $\quad \lim_{x\to 2}\left(\frac{x^2-4}{x^2+x-6}\right) = \lim_{x\to 2}\left(\frac{\cancel{(x-2)}(x+2)}{(x+3)\cancel{(x-2)}}\right) = \frac{4}{5}$

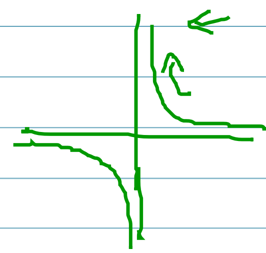
x	f(x)
1.9	0.7959
1.99	0.799599
1.999	0.79995999
	0.8

x	f(x)
2.1	0.8039
2.01	0.800399
2.001	0.800039992
	0.8

Infinite Limit

Let f be a function defined on both sides of a, except possibly at a. $\lim_{x\to a}(f(x)) = \pm\infty$ means that the values of $f(x)$ can be made arbitrarily large (or large negative) by taking x sufficiently close to a, but not equal to a





Vertical Asymptote $- \ x = a$

$\lim_{x\to a^-}(f(x)) = \pm\infty$ $\quad \lim_{x\to a^+}(f(x)) = \pm\infty$ $\quad \lim_{x\to a}(f(x)) = \pm\infty$

$\lim_{x\to 0}\left(\frac{1}{x}\right) = DNE$ $\quad \lim_{x\to 0^-}\left(\frac{1}{x}\right) = -\infty$ $\quad \lim_{x\to 0^+}\left(\frac{1}{x}\right) = \infty$