Logarithms

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$$y = \ln(x)$$
$$e^y = x$$
$$e^y \frac{dy}{dx} = 1$$
$$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a(x)) = \frac{d}{dx}\left(\frac{\ln(x)}{\ln(a)}\right)$$

$$\frac{1}{\ln(a)} \frac{d}{dx}(\ln(x)) = \frac{1}{\ln(a)} \frac{1}{x}$$

$$y = \log_a(x)$$
$$a^y = x$$
$$a^y \ln(a) \frac{dy}{dx} = 1 \qquad \frac{dy}{dx} = \frac{1}{\ln(a) a^y} = \frac{1}{x \ln(a)}$$

$$\rightarrow \frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln(a)}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x \ln(e)} = \frac{1}{x} \leftarrow$$

$$\frac{d}{dx}(\ln(u)) = \frac{1}{u} u' = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx}(\ln(g(x))) = \frac{1}{g(x)} g'(x)$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$$

Logarithmic Differentiation

1) Take ln of both sides
2) simplify using properties of logs
3) use implicit differentiation to take the derivative
4) solve for $y' = \frac{dy}{dx}$

$$f(x) = x^n = y$$
$$\ln(x^n) = \ln(y)$$
$$n \ln(x) =$$
$$n \frac{1}{x} = \frac{1}{y} y' \leftarrow$$
$$n \frac{y}{x} = y' \leftarrow$$

$$y' = n \frac{x^n}{x} = n x^{n-1}$$

Logarithms

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

$y = \ln(x)$
$e^y = x$
$e^y \frac{dy}{dx} = 1$
$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{e^{\ln(x)}} = \frac{1}{x}$

$$\frac{d}{dx}(\log_a(x)) = \frac{d}{dx}\left(\frac{\ln(x)}{\ln(a)}\right)$$

$$\frac{1}{\ln(a)} \frac{d}{dx}(\ln(x)) = \frac{1}{\ln(a)} \frac{1}{x}$$

$y = \log_a(x)$
$a^y = x$
$a^y \ln(a) \frac{dy}{dx} = 1$ $\quad \frac{dy}{dx} = \frac{1}{\ln(a) a^y} = \frac{1}{x \ln(a)}$

$$\longrightarrow \frac{d}{dx}\left(\log_a(x)\right) = \frac{1}{x\ln(a)}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x\ln(e)} = \frac{1}{x} \leftarrow$$

$$\frac{d}{dx}\left(\ln(u)\right) = \frac{1}{u} u' = \frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}\left(\ln(g(x))\right) = \frac{1}{g(x)} g'(x)$$

$$\frac{d}{dx}\left(\ln|x|\right) = \frac{1}{x}$$

Logarithmic Differentiation

1) Take ln of both sides
2) simplify using properties of logs
3) use implicit differentiation to take the derivative
4) solve for $y' = \frac{dy}{dx}$

$f(x) = x^n = y \leftarrow$
$\ln(x^n) = \ln(y)$
$n\ln(x) =$
$n \frac{1}{x} = \frac{1}{y} y' \leftarrow$
$n \frac{y}{x} = y' \leftarrow$

$$y' = n\frac{x^n}{x} = nx^{n-1}$$

$y = a^x$

$\ln(y) = \ln(a^x) = x\ln(a)$

$\frac{1}{y} y' = \ln(a)$

$y' = y\ln(a) = a^x \ln(a)$

$y = x^{\sqrt{x}}$

$\ln(y) = \ln\left(x^{\sqrt{x}}\right) = \sqrt{x}\,\ln(x)$

$\frac{1}{y} y' = \sqrt{x}\,\frac{1}{x} + \frac{1}{2} x^{-1/2} \ln(x)$

$= \frac{1}{\sqrt{x}} + \frac{\ln(x)}{2\sqrt{x}} = \frac{2 + \ln(x)}{2\sqrt{x}}$

$y' = \left(\frac{2+\ln(x)}{2\sqrt{x}}\right) y = \left(\frac{2+\ln(x)}{2\sqrt{x}}\right) x^{\sqrt{x}} \leftarrow$

$y = (3x-5)^3 (9x+1)^5$

$y' = 3(3x-5)^2 3(9x+1)^5 + (3x-5)^3\, 5(9x+1)^4 9 = 9(3x-5)^2(9x+1)^5 + 45(3x-5)^3(9x+1)^4$

$\ln(y) = \ln\left((3x-5)^3(9x+1)^5\right) = 3\ln(3x-5) + 5\ln(9x+1)$

$\frac{1}{y} y' = \frac{3}{3x-5}\cdot(3) + \frac{5}{9x+1}(9)$

$y' = \left(\frac{9}{3x-5} + \frac{45}{9x+1}\right)(3x-5)^3(9x+1)^5$

---

e as a limit

$f(x) = \ln(x)$

$f'(x) = \frac{1}{x}$

$f'(1) = 1 = \lim_{h\to 0}\left(\frac{f(1+h) - f(1)}{h}\right)$

$= \lim_{h\to 0}\left(\frac{\ln(1+h) - \ln(1)}{h}\right)$

$= \lim_{h\to 0}\left(\frac{1}{h}\ln(1+h)\right)$

$= \lim_{h\to 0}\left(\ln(1+h)^{1/h}\right)$

$e^1 = e^{\lim_{h\to 0}\left(\ln\left((1+h)^{1/h}\right)\right)}$

$e = \lim_{h\to 0}\left((1+h)^{1/h}\right)$

$e = \lim_{x\to 0}\left((1+x)^{1/x}\right) \leftarrow$

$= \lim_{x\to\infty}\left(\left(1+\frac{1}{x}\right)^x\right) \leftarrow$