Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$

$f(x) = 4x^2-3x+7$

$f(x+h) = 4(x+h)^2-3(x+h)+7$

$4x^2-3x+7+h = f(x)+h$

$$\frac{4(x+h)^2-3(x+h)+7-(4x^2-3x+7)}{h}$$

$$\frac{4(x^2+2xh+h^2)-3x-3h+7-4x^2+3x-7}{h}$$

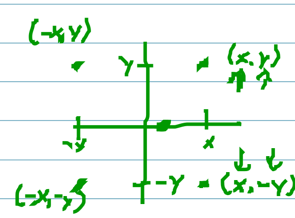
$$\frac{4x^2+8xh+4h^2-3x-3h+7-4x^2+3x-7}{h}$$

$$\Rightarrow \frac{8xh+4h^2-3h}{h} = \frac{\cancel{h}(8x+4h-3)}{\cancel{h}} = \boxed{8x-4h+3}$$

Symmetry

y-axis – Even

$\rightarrow f(-x) = f(x)$



origin – Odd

$\rightarrow -f(-x) = f(x) \rightarrow f(-x) = -f(x)$

x-axis –

$f(x) = 7x^2-5$

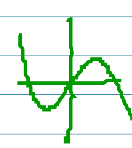
$f(-x) = 7(-x)^2-5 = 7x^2-5$ Even

$-f(x) = -(7x^2-5) = -7x^2+5$ Not odd

$f(x) = -3x^5+4x^3$ Not Even

$f(-x) = -3(-x)^5+4(-x)^3 = 3x^5-4x^3$

$-f(x) = -(-3x^5+4x^3) = 3x^5-4x^3$ Odd



Difference Quotent

$$\frac{f(x+h)-f(x)}{h}$$

$f(x) = 4x^2 - 3x + 7$ $\qquad\qquad$ $\underline{4x^2-3x+7}+h = f(x)+h$

$f(x+h) = 4(x+h)^2 - 3(x+h) + 7$

$$\frac{4(x+h)^2-3(x+h)+7-(4x^2-3x+7)}{h}$$

$$\frac{4(x^2+2xh+h^2)-3x-3h+7-4x^2+3x-7}{h}$$

$$\frac{4x^2+8xh+4h^2-3x-3h+7-4x^2+3x-7}{h}$$

$$\Rightarrow \frac{8xh+4h^2-3h}{h} = \frac{\cancel{h}(8x+4h-3)}{\cancel{h}} = \boxed{8x-4h+3}$$

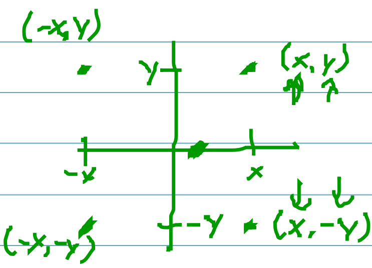
Symmetry

y-axis – Even

$\Rightarrow f(-x) = f(x) \Leftarrow$

origin – Odd

$\Rightarrow -f(-x) = f(x) \Rightarrow f(-x) = -f(x) \Leftarrow$

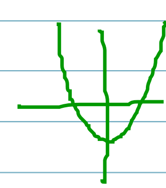


~~x-axis~~ –

$f(x) = 7x^2 - 5$

$f(-x) = 7(-x)^2 - 5 = 7x^2 - 5$ $\qquad$ Even

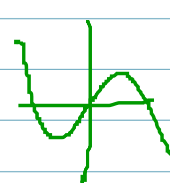
$-f(x) = -(7x^2-5) = -7x^2+5$ $\qquad$ Not odd



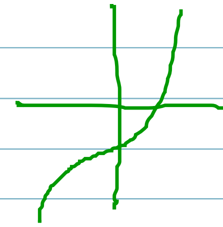
$f(x) = -3x^5 + 4x^3$ $\qquad$ Not Even

$f(-x) = -3(-x)^5 + 4(-x)^3 = 3x^5 - 4x^3$

$-f(x) = -(-3x^5+4x^3) = 3x^5 - 4x^3$ $\qquad$ odd



$f(x) = 4x^3 - 5$



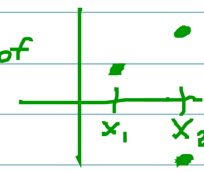
$f(-x) = 4(-x)^3 - 5 = -4x^3 - 5$

$-f(x) = -(4x^3 - 5) = -4x^3 + 5$

Direction $\quad I =$ open interval

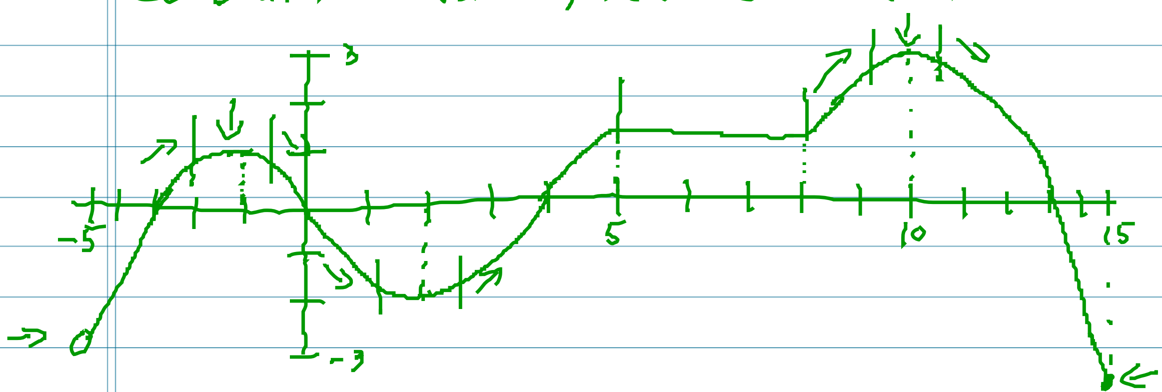
Increase – If $x_1 < x_2$, Then $f(x_1) < f(x_2)$ $\quad \forall x_1, x_2 \in I$

$\forall \Rightarrow$ For all $\quad \in \rightarrow$ an element of



Decrease – If $x_1 < x_2$, Then $f(x_1) > f(x_2)$ $\quad \forall x_1, x_2 \in I$

Constant – $\forall x \in I$, $f(x) = c =$ constant



Inc – $(-5, -1) \cup (2, 5) \cup (8, 10)$

Dec – $(-1, 2) \cup (10, 15)$

Con – $(5, 8)$

local Maximum – $c \in I$ st $\forall x \in I$ $f(c) \geq f(x)$

local minumum – $c \in I$ st $\forall x \in I$ $f(c) \leq f(x)$

Absolute Maximum – $c \in D$ st $\forall x \in D$ $f(c) \geq f(x)$

Absolute Minimum – $c \in D$ st $\forall x \in D$ $f(c) \leq f(x)$

Exetrema

Local minimum – $[5, 8]$

Local maximum – $[5, 8]$

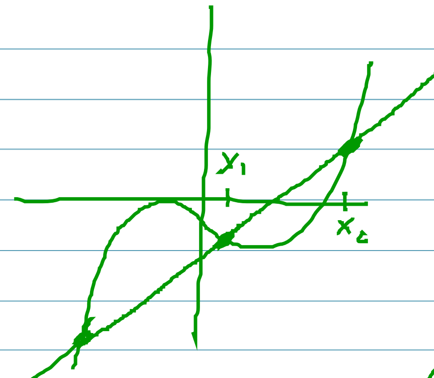
Extreme Value Theorem – EVT

If f = continuous function on $[a, b]$, Then f has an absolute minimum and an absolute maximum on $[a, b]$

Average Rate of Change = slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

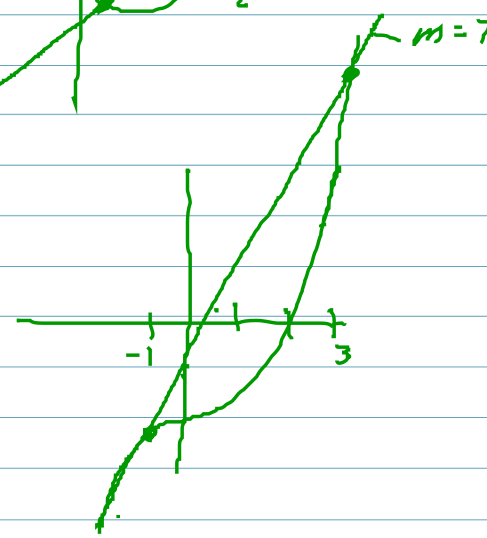
secant line



$f(x) = x^3 - 8$
$[-1, 3]$

$f(-1) = -9$
$f(3) = 19$

$$m_{sec} = \frac{19 - -9}{3 - -1} = \frac{19 + 9}{3 + 1} = \frac{28}{4} = 7$$



$$\frac{f(x+h) - f(x)}{x + h - x} = m_{sec}$$

$$\rightarrow \frac{f(x+h) - f(x)}{h} =$$