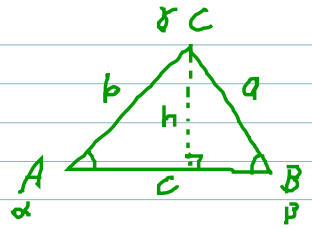
Oblique



AAA ✗
ASA ✓
SAS ✗
~~ASS~~ SSA ✓
AAS ✓
SSS ✗

$\sin(A) = \frac{h}{b}$ $\sin(B) = \frac{h}{a}$

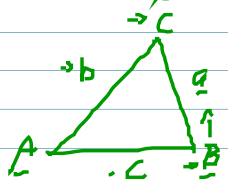
$b\sin(A) = h = a\sin(B)$

Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

AAS



$\rightarrow m\angle A = 25^\circ$ $m\angle B = 50^\circ$ $\rightarrow a = 10$

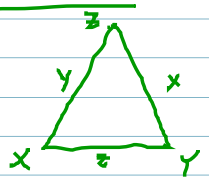
$m\angle C = 105^\circ$
$b = 18.126$
$c = 22.856$

$$\frac{10}{\sin(25^\circ)} = \frac{b}{\sin(50^\circ)} \qquad b = \frac{10\sin(50^\circ)}{\sin(25^\circ)} \approx 18.126155574$$

$$\frac{10}{\sin(25^\circ)} = \frac{c}{\sin(105^\circ)} \qquad c = \frac{10\sin(105^\circ)}{\sin(25^\circ)} \approx 22.855752194$$

Scalene Inequality

ASA

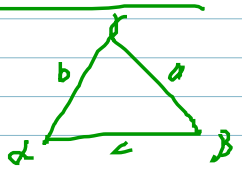


$m\angle X = 65^\circ$ $m\angle Y = 80^\circ$ $z = 15$

$m\angle z = 35^\circ$
$x = 23.701$
$y = 25.754$

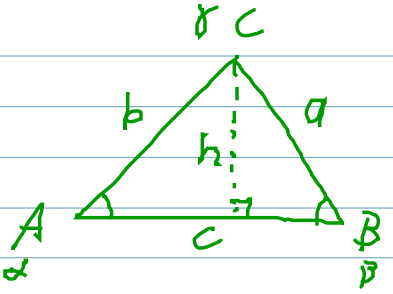
$$\frac{X}{\sin(65^\circ)} = \frac{15}{\sin(35^\circ)} \qquad \frac{Y}{\sin(80^\circ)} = \frac{15}{\sin(35^\circ)}$$

AAA



$m\angle\alpha = 50^\circ$
$m\angle\beta = 85^\circ$
$m\angle\gamma = 45^\circ$

Oblique



AAA X
ASA ✓
SAS X
~~ASS~~ SSA ✓
AAS ✓
SSS X

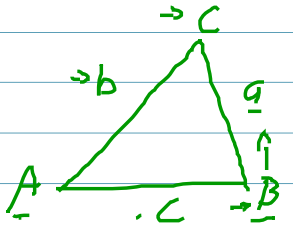
$\sin(A) = \frac{h}{b}$ $\sin(B) = \frac{h}{a}$

$b\sin(A) = h = a\sin(B)$

Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c} \qquad \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

AAS



→ $m\angle A = 25°$ >
$m\angle B = 50°$
→ $a = 10$

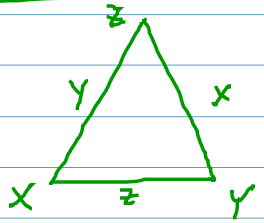
$m\angle C = 105°$
$b = 18.126$
$c = 22.856$

$$\frac{10}{\sin(25°)} = \frac{b}{\sin(50°)} \qquad b = \frac{10\sin(50°)}{\sin(25°)} = 18.126155\overline{74}$$

$$\frac{10}{\sin(25°)} = \frac{c}{\sin(105°)} \qquad c = \frac{10\sin(105°)}{\sin(25°)} = 22.85575\overline{219}$$

Scalene Inequality

ASA

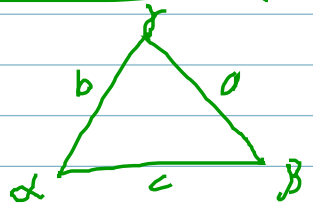


$m\angle X = 65°$ >
$m\angle Y = 80°$
$z = 15$

$m\angle Z = 35°$
$x = 23.701$
$y = 25.754$

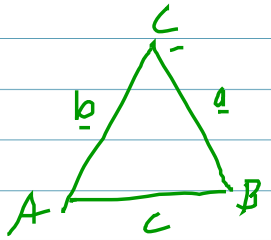
$$\frac{X}{\sin(65°)} = \frac{15}{\sin(35°)} \qquad \frac{Y}{\sin(80°)} = \frac{15}{\sin(35°)}$$

AAA



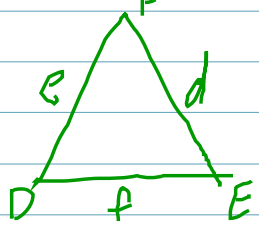
$m\angle\alpha = 50°$
$m\angle\beta = 85°$
$m\angle\gamma = 45°$

SAS



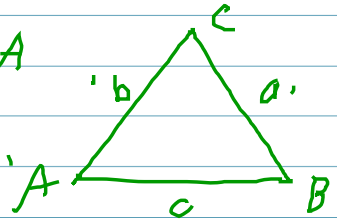
$a = 12$
$b = 8$
$m\angle C = 55°$

SSS



$d = 8$
$e = 15$
$f = 10$

~~ASS~~ SSA



$m\angle A = 30°$
$a = 10$
$b = 12$

$m\angle B = 36.87°$
$m\angle C = 113.13°$
$c = 18.392$

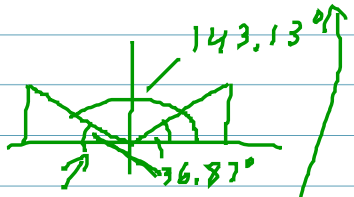
$$\frac{\sin(B)}{12} = \frac{\sin(30°)}{10}$$

$$\sin(B) = \frac{12\sin(30°)}{10}$$

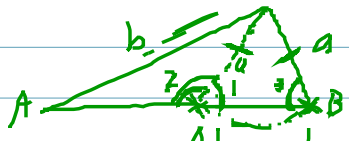
$$m\angle B = \sin^{-1}(0.6) = 36.869897645°$$

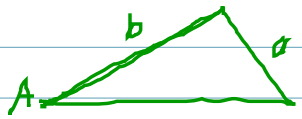
$m\angle C = 113.13010235°$

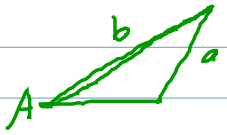
$$\frac{c}{\sin(113.13010235°)} = \frac{10}{\sin(30°)}$$



Ambiguous





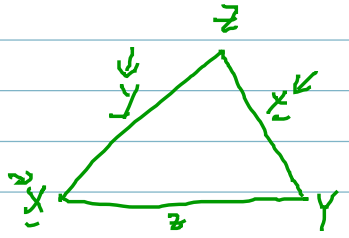


143.13010234

	Δ1	Δ2
$m\angle A$	30°	30°
$m\angle B$	36.87°	143.13° ⇐ 180 − $m\angle B$
$m\angle C$	113.13°	6.87° ⇐ 6.869897645
a	10	10
b	12	12
c	18.392	2.392

$$\frac{c}{\sin(6.87°)} = \frac{10}{\sin(30°)}$$

→ $m\angle X = 40°$
→ $x = 15$
→ $y = 16$

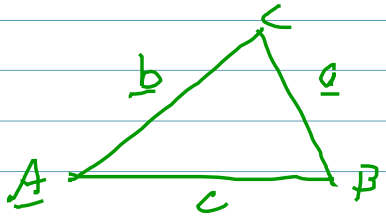


	Δ_1	Δ_2
$m\angle X$	40°	40°
$m\angle Y$	43,286°	136,714°
$m\angle Z$	96,714°	3,286°
x	15	15
y	16	16
z	23,176	~~0,937~~ 1,338

$\frac{\sin(Y)}{16} = \frac{\sin(40°)}{15}$

$\frac{z}{\sin(96,714°)} = \frac{15}{\sin(40°)}$ $\quad$ $\frac{z}{\sin(3,286°)} = \frac{15}{\sin(40°)}$

→ $m\angle A = 80°$
$a = 3$
→ $b = 15$

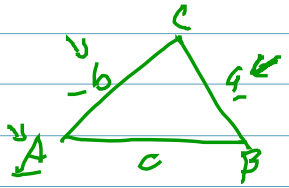


	$\Delta 1$	$\Delta 2$
$m\angle A$	80°	80°
$m\angle B$		
$m\angle C$		
a	3	3
b	15	15
c		

$\frac{\sin(B)}{15} = \frac{\sin(80°)}{3}$

$\sin(B) = \frac{15\sin(80°)}{3} = 4.924$

$m\angle A = 65°$
$a = 18$
$b = 12$

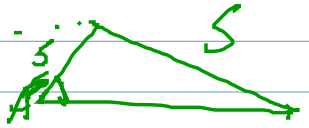


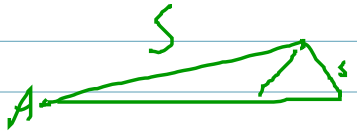
	$\Delta 1$	$\Delta 2$
$m\angle A$	65°	65°
$m\angle B$	37,172°	142,828°
$m\angle C$	77,828°	
a	18	18
b	12	12
c	19,44	

$\frac{\sin(B)}{12} = \frac{\sin(65°)}{18}$

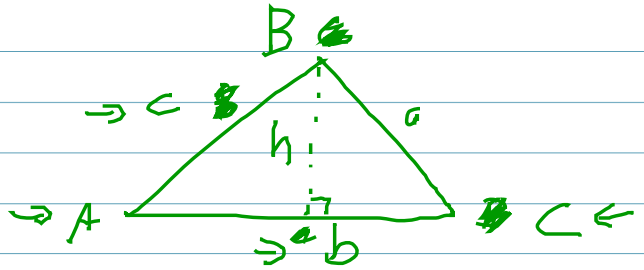
$\frac{c}{\sin(77,828°)} = \frac{18}{\sin(65°)}$

SsA





$A = \frac{1}{2}bh$



SAS

$\sin(A) = \frac{h}{c} \Rightarrow c\sin(A) = h$

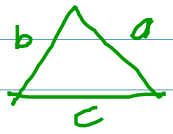
$\Rightarrow A = \frac{1}{2}bc\sin(A)$ $\quad A = \frac{1}{2}ac\sin(B)$ $\quad A = \frac{1}{2}ab\sin(C)$

Heron's Formula

SSS

$A = \sqrt{s(s-a)(s-b)(s-c)}$

semi-perimeter

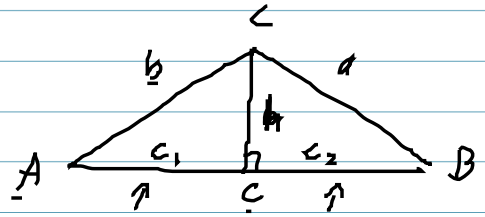


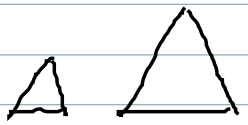
$\frac{a+b+c}{2} = s$

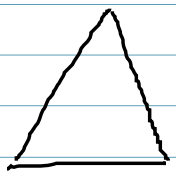
AAS ✓ $\quad$ AAA ✗

ASA ✓ $\quad$ SAS ✓

SSA ✓ $\quad$ SSS ✓







$\Rightarrow \cos(A) = \frac{c_1}{b}$

$\Rightarrow c_1^2 + h^2 = b^2$

$c_2^2 + h^2 = a^2$

$\Rightarrow c_1 + c_2 = c$

$a^2 = c_2^2 + h^2$

$= (c - c_1)^2 + b^2 - c_1^2$

$= c^2 - 2cc_1 + c_1^2 + b^2 - c_1^2$

$= b^2 + c^2 - 2cc_1$

$= b^2 + c^2 - 2cb\cos(A)$

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$2bc\cos(A) = b^2 + c^2 - a^2$

$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$ SSS

SAS

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$b^2 = a^2 + c^2 - 2ac\cos(B)$

$c^2 = a^2 + b^2 - 2ab\cos(C)$

$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$