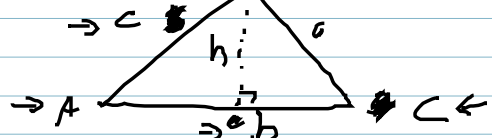


$$A = \frac{1}{2}bh$$



$$\sin(A) = \frac{h}{c} \rightarrow c \sin(A) = h$$

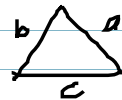
$$\rightarrow A = \frac{1}{2}bc \sin(A) \quad A = \frac{1}{2}ac \sin(B) \quad A = \frac{1}{2}ab \sin(C)$$

Heron's Formula

SSS

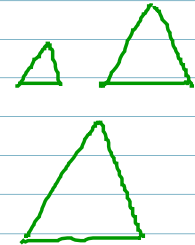
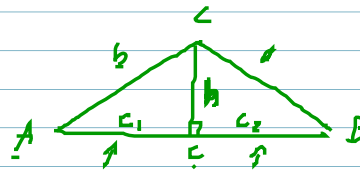
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

semi-perimeter



$$\frac{a+b+c}{2} = s$$

- AAS ✓
- ASA ✓
- SSA ✓
- AAA ✗
- SAS ✓
- SSS ✓



$$\rightarrow \cos(A) = \frac{c_1}{b}$$

$$\rightarrow c_1^2 + h^2 = b^2$$

$$c_2^2 + h^2 = a^2$$

$$\rightarrow c_1 + c_2 = c$$

$$a^2 = c_2^2 + h^2$$

$$= (c - c_1)^2 + b^2 - c_1^2$$

$$= c^2 - 2cc_1 + c_1^2 + b^2 - c_1^2$$

$$= b^2 + c^2 - 2cc_1$$

$$= b^2 + c^2 - 2cb \cos(A)$$

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

SAS

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$2bc \cos(A) = b^2 + c^2 - a^2$$

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

SSS

$A = \frac{1}{2}bh$
 $\rightarrow c$
 $\rightarrow A$
 $\rightarrow b$
 $\rightarrow C$
SAS

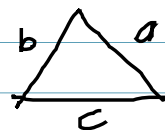
$\sin(A) = \frac{h}{c} \rightarrow c \sin(A) = h$

$\Rightarrow A = \frac{1}{2}bc \sin(A) \quad A = \frac{1}{2}ac \sin(B) \quad A = \frac{1}{2}ab \sin(C)$

Heron's Formula

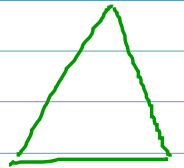
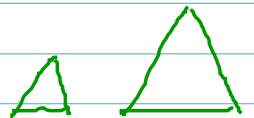
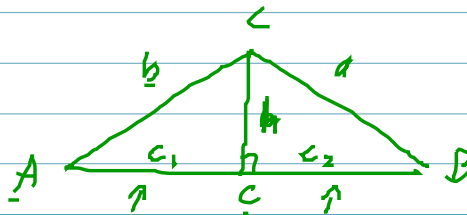
$A = \sqrt{s(s-a)(s-b)(s-c)}$

semi-perimeter



$\frac{a+b+c}{2} = s$

- | | |
|-------|-------|
| AAS ✓ | AAA ✗ |
| ASA ✓ | SAS ✓ |
| SSA ✓ | SSS ✓ |



$\rightarrow \cos(A) = \frac{c_1}{b}$
 $\rightarrow c_1^2 + h^2 = b^2$
 $c_2^2 + h^2 = a^2$
 $\rightarrow c_1 + c_2 = c$

$a^2 = c_2^2 + h^2$
 $= (c - c_1)^2 + b^2 - c_1^2$
 $= c^2 - 2cc_1 + \underline{c_1^2} + b^2 - \underline{c_1^2}$
 $= b^2 + c^2 - 2cc_1$
 $= b^2 + c^2 - 2cb \cos(A)$

$a^2 = b^2 + c^2 - 2bc \cos(A)$

SAS

$2bc \cos(A) = b^2 + c^2 - a^2$

$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$

SSS

$a^2 = b^2 + c^2 - 2bc \cos(A)$

$b^2 = a^2 + c^2 - 2ac \cos(B)$

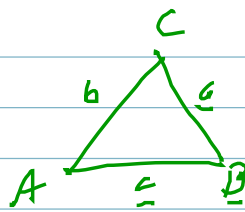
$c^2 = a^2 + b^2 - 2ab \cos(C)$

$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$

$m\angle B = 55^\circ$
 $a = 12$
 $c = 15$

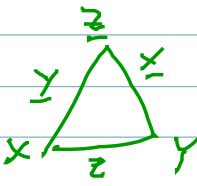


$b^2 = 12^2 + 15^2 - 2(12)(15)\cos(55^\circ)$
 $= 162.5124829$
 $b = 12.74803839$

$m\angle A = 50.451^\circ$
 $m\angle C = 74.549^\circ$

$\frac{\sin(A)}{12} = \frac{\sin(55^\circ)}{12.74803839}$

$m\angle Z = 70^\circ$
 $x = 20$
 $y = 15$



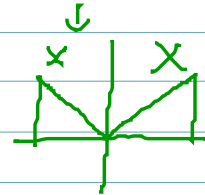
$z^2 = 20^2 + 15^2 - 2(20)(15)\cos(70^\circ)$
 $= 419.787914$
 $z = 20.48872651$

$m\angle X = 66.531^\circ$
 $m\angle Y = 43.469^\circ$

$\frac{\sin(X)}{20} = \frac{\sin(70^\circ)}{20.48872651}$

$a = 10$
 $b = 12$
 $c = 20$

$\cos(A) = \frac{12^2 + 20^2 - 10^2}{2(12)(20)}$
 $m\angle A = 22.332^\circ$



$\frac{\sin(C)}{20} = \frac{\sin(22.332^\circ)}{10}$

$m\angle C = 49.459^\circ$

$m\angle B = 108.209^\circ$


$\frac{\sin(B)}{12} = \frac{\sin(22.332^\circ)}{10}$

$m\angle B = 27.127^\circ$
 $m\angle C = 130.541^\circ$

$\cos(C) = \frac{10^2 + 12^2 - 20^2}{2(10)(12)}$

$a = 5$
 $b = 6$
 $c = 15$

$\cos(c) = \frac{5^2 + 6^2 - 15^2}{2(5)(6)} = -2.73$



Triangle Inequality

$a \leq b + c$
 $b \leq a + c$
 $c \leq a + b$

$c^2 = a^2 + b^2 - \frac{2ab\cos(c)}{1}$

$c = 90^\circ \rightarrow c^2 = a^2 + b^2$

$0^\circ < c < 90^\circ \rightarrow c^2 = a^2 + b^2 - \text{something}$

$90^\circ < c < 180^\circ \rightarrow c^2 = a^2 + b^2 + \text{something}$

