Hyperbolic Functions

$\sinh(x) = \frac{e^x - e^{-x}}{2}$

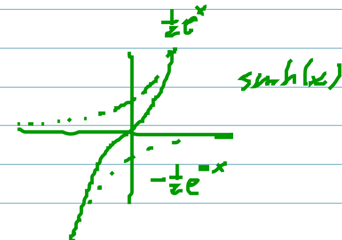
$\cosh(x) = \frac{e^x + e^{-x}}{2}$

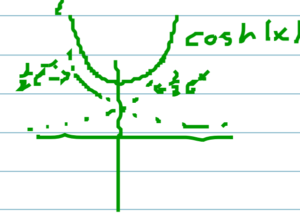
$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

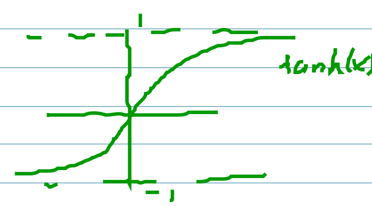
$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}$

$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}$

$\coth(x) = \frac{1}{\tanh(x)} = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$







Identities

$\sinh(-x) = -\sinh(x)$ $\qquad$ $\cosh(-x) = \cosh(x)$

$\cosh^2(x) - \sinh^2(x) = 1$

$1 - \tanh^2(x) = \operatorname{sech}^2(x)$

$\coth^2(x) - 1 = \operatorname{csch}^2(x)$

$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$

$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$

$f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2}$ $\qquad$ $g(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$

$f'(x) = \frac{e^x + e^{-x}}{2} = \cosh(x)$ $\qquad$ $g'(x) = \frac{e^x - e^{-x}}{2} = \sinh(x)$

$\frac{d}{dx}(\tanh(x)) = \operatorname{sech}^2(x)$ $\qquad$ $\frac{d}{dx}(\operatorname{sech}(x)) = -\operatorname{sech}(x)\tanh(x)$

Hyperbolic Functions

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

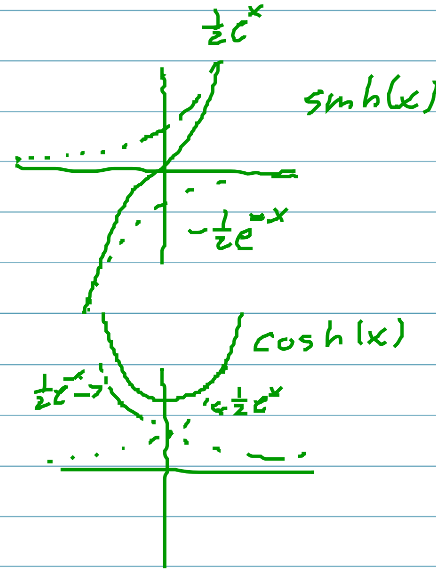
$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

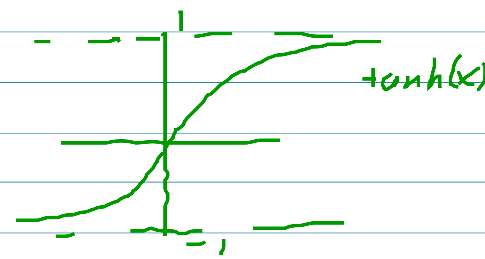
$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} = \frac{2}{e^x - e^{-x}}$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} = \frac{2}{e^x + e^{-x}}$$

$$\coth(x) = \frac{1}{\tanh(x)} = \frac{\cosh(x)}{\sinh(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$





Identities

$\sinh(-x) = -\sinh(x)$ $\qquad$ $\cosh(-x) = \cosh(x)$

$$\cosh^2(x) - \sinh^2(x) = 1$$
$$1 - \tanh^2(x) = \operatorname{sech}^2(x)$$
$$\coth^2(x) - 1 = \operatorname{csch}^2(x)$$

$$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$
$$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

$f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2}$ $\qquad$ $g(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$

$f'(x) = \frac{e^x + e^{-x}}{2} = \cosh(x)$ $\qquad$ $g'(x) = \frac{e^x - e^{-x}}{2} = \sinh(x)$

$\frac{d}{dx}\left(\tanh(x)\right) = \operatorname{sech}^2(x)$ $\qquad$ $\frac{d}{dx}\left(\operatorname{sech}(x)\right) = -\operatorname{sech}(x)\tanh(x)$

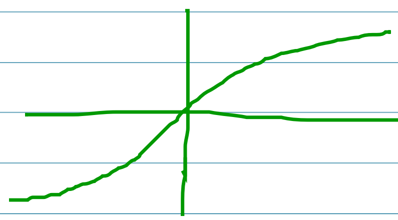
$$\frac{d}{dx}\left(\coth(x)\right) = -\operatorname{csch}^2(x) \qquad \frac{d}{dx}\left(\operatorname{csch}(x)\right) = -\operatorname{csch}(x)\coth$$

$\sinh^{-1}(x) = y$ iff $\sinh(y) = x$

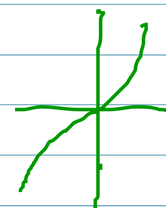
$\cosh^{-1}(x) = y$ iff $\cosh(y) = x$, $y \geq 0$

$\tanh^{-1}(x) = y$ iff $\tanh(y) = x$

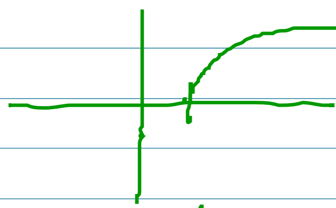
$$\sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right), \quad x \in \mathbb{R}$$

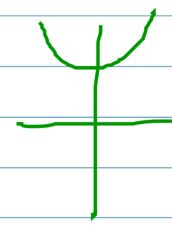


$D \in \mathbb{R}$
$R \in \mathbb{R}$

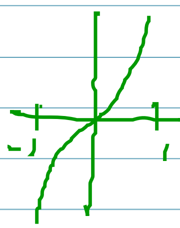


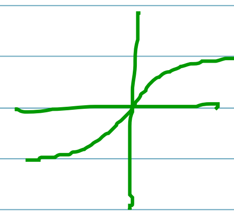
$$\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2-1}\right), \quad x \in [1, \infty)$$





$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right), \quad x \in (-1, 1)$$





$$\frac{d}{dx}\left(\sinh^{-1}(x)\right) = \frac{1}{x+\sqrt{x^2+1}}\left(1 + \frac{2x}{2\sqrt{x^2+1}}\right) = \frac{\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}}{x+\sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}\left(\cosh^{-1}(x)\right) = \frac{1}{\sqrt{x^2-1}} \qquad \frac{d}{dx}\left(\coth^{-1}(x)\right) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}\left(\tanh^{-1}(x)\right) = \frac{1}{1-x^2} \qquad \frac{d}{dx}\left(\operatorname{sech}^{-1}(x)\right) = -\frac{1}{x\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\operatorname{csch}^{-1}(x)\right) = \frac{1}{|x|\sqrt{x^2+1}}$$