Indeterminant Form

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{0}{0} \quad \pm\frac{\infty}{\infty}$$

$\lim_{x\to 0}\left(\frac{x^2}{x}\right) = \frac{0}{0} \to \lim_{x\to 0}(x) = 0$ $\qquad \lim_{x\to 0}\left(\frac{\frac{1}{x}}{\frac{1}{x^2}}\right) = 0$

$\lim_{x\to 0}\left(\frac{x}{x^3}\right) = \frac{0}{0} \to \lim_{x\to 0}\left(\frac{1}{x^2}\right) = \infty$

$\to \lim_{x\to 0}\left(\frac{3x}{x}\right) = \frac{0}{0} \to \lim_{x\to 0}(3) = 3$

L'Hospital's Rule

If f, g = differentiable, $g'(x) \neq 0$ for x near a,
$\lim_{x\to a} f(x) = 0$ + $\lim_{x\to a} g(x) = 0$ OR $\lim_{x\to a} f(x) = \pm\infty$ + $\lim_{x\to a} g(x) = \pm\infty$

Then $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$, if $\lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$ Exists or $= \pm\infty$

$\lim_{x\to\infty}\left(\frac{e^x}{x^2}\right) = \frac{\infty}{\infty} \to \lim_{x\to\infty}\left(\frac{e^x}{2x}\right) = \frac{\infty}{\infty} \to \lim_{x\to\infty}\left(\frac{e^x}{2}\right) = \infty$

$\lim_{x\to 0}\left(\frac{e^x - 1}{x^3}\right) = \frac{0}{0} \to \lim_{x\to 0}\left(\frac{e^x}{3x^2}\right) = \frac{1}{0^+} = \infty$

$\lim_{x\to 0}\left(\frac{2\sin(x)}{x}\right) = \frac{0}{0} \to \lim_{x\to 0}\left(\frac{2\cos(x)}{1}\right) = 2$

$\infty - \infty \qquad (0)(\infty) \qquad 1^\infty$

$\lim_{x\to 0^+}\left(x^2\ln(x)\right) = (0)(-\infty) \to \lim_{x\to 0^+}\left(\frac{\ln(x)}{\frac{1}{x^2}}\right) = \frac{-\infty}{\infty}$

$\to \lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{-\frac{2}{x^3}}\right) = \lim_{x\to 0^+}\left(-\frac{x^3}{2x}\right) = \lim_{x\to 0^+}\left(-\frac{x^2}{2}\right) = 0$

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Indeterminant Form

$$\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{0}{0} \qquad \pm\frac{\infty}{\infty}$$

$$\lim_{x\to 0}\left(\frac{x^2}{x}\right) = \frac{0}{0} \Rightarrow \lim_{x\to 0}(x) = 0 \qquad \lim_{x\to 0}\left(\frac{\frac{1}{x}}{\frac{1}{x^2}}\right) = 0$$

$$\lim_{x\to 0}\left(\frac{x}{x^3}\right) = \frac{0}{0} \longrightarrow \lim_{x\to 0}\left(\frac{1}{x^2}\right) = \infty$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{3x}{x}\right) = \frac{0}{0} \longrightarrow \lim_{x\to 0}(3) = 3$$

L'Hospital's Rule

If f, g = differentiable, $g'(x) \neq 0$ for x near a,
$\lim_{x\to a} f(x) = 0$ + $\lim_{x\to a} g(x) = 0$ OR $\lim_{x\to a} f(x) = \pm\infty$ + $\lim_{x\to a} g(x) = \pm\infty$

Then $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$, if $\lim_{x\to a}\left(\frac{f'(x)}{g'(x)}\right)$ Exists or $= \pm\infty$

$$\lim_{x\to\infty}\left(\frac{e^x}{x^2}\right) = \frac{\infty}{\infty} \longrightarrow \lim_{x\to\infty}\left(\frac{e^x}{2x}\right) = \frac{\infty}{\infty} \longrightarrow \lim_{x\to\infty}\left(\frac{e^x}{2}\right) = \infty$$

$$\lim_{x\to 0}\left(\frac{e^x - 1}{x^3}\right) = \frac{0}{0} \Rightarrow \lim_{x\to 0}\left(\frac{e^x}{3x^2}\right) = \frac{1}{0^+} = \infty$$

$$\lim_{x\to 0}\left(\frac{2\sin(x)}{x}\right) = \frac{0}{0} \Rightarrow \lim_{x\to 0}\left(\frac{2\cos(x)}{1}\right) = 2$$

$$\infty - \infty \qquad (0)(\infty) \qquad 1^\infty$$

$$\lim_{x\to 0^+}\left(x^2\ln(x)\right) = (0)(-\infty) \Rightarrow \lim_{x\to 0^+}\left(\frac{\ln(x)}{\frac{1}{x^2}}\right) = \frac{-\infty}{\infty}$$

$$\Rightarrow \lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{-\frac{2}{x^3}}\right) = \lim_{x\to 0^+}\left(-\frac{x^3}{2x}\right) = \lim_{x\to 0^+}\left(-\frac{x^2}{2}\right) = 0$$

$$\lim_{x\to 1^+}\left(\frac{1}{\ln(x)}-\frac{1}{x-1}\right) = \infty - \infty$$

$$\lim_{x\to 1^+}\left(\frac{x-1-\ln(x)}{(x-1)\ln(x)}\right) = \lim_{x\to 1^+}\left(\frac{1-\frac{1}{x}}{(x-1)\frac{1}{x}+\ln(x)}\right) = \lim_{x\to 1^+}\left(\frac{\frac{1}{x^2}}{\frac{1}{x^2}+\frac{1}{x}}\right) = \frac{1}{2}$$

$$\frac{x-1}{x}+\ln(x)$$

$$1-\frac{1}{x}+\ln(x)$$

$$\lim_{x\to\infty}(x-e^x) = \lim_{x\to\infty}\left(x\left(1-\frac{e^x}{x}\right)\right) = \infty(1-\infty) = \infty(-\infty) = -\infty$$

$$\lim_{x\to\infty}\left(\frac{e^x}{x}\right) = \lim_{x\to\infty}\left(\frac{e^x}{1}\right) = \infty$$

$$\lim_{x\to\infty}(x^3-x) = \lim_{x\to\infty}\left(x(x^2-1)\right) = \infty(\infty^2-1) = \infty(\infty) = \infty$$

$1^\infty \quad 0^0 \quad \infty^0$

$$\lim_{x\to 0^+}\left(\frac{1}{x}\right)^{\sin(x)} = \lim_{x\to 0^+}\left[e^{\ln\left(\left(\frac{1}{x}\right)^{\sin(x)}\right)}\right] = e^0 = 1$$

$$\lim_{x\to 0^+}\left(\ln\left(\left(\frac{1}{x}\right)^{\sin(x)}\right)\right) = \lim_{x\to 0^+}\left(\sin(x)\ln\left(\frac{1}{x}\right)\right) = \lim_{x\to 0^+}\left(\frac{\ln\left(\frac{1}{x}\right)}{\frac{1}{\sin(x)}}\right) = \lim_{x\to 0^+}\left(\frac{\ln\left(\frac{1}{x}\right)}{\csc(x)}\right)$$

$$= \lim_{x\to 0^+}\left(\frac{\frac{1}{1/x}\left(-\frac{1}{x^2}\right)}{-\csc(x)\cot(x)}\right) = \lim_{x\to 0^+}\frac{\left(-\frac{1}{x^2}\cdot\frac{x}{1}\right)}{-\frac{1}{\sin(x)}\cdot\frac{\cos(x)}{\sin(x)}} = \lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{\frac{\cos(x)}{\sin^2(x)}}\right)$$

$$= \lim_{x\to 0^+}\left(\frac{\sin^2(x)}{x\cos(x)}\right) = \lim_{x\to 0^+}\left(\frac{2\sin(x)\cos(x)}{\cos(x)-x\sin(x)}\right) = \frac{0}{1} = 0$$

$$\lim_{x\to 0^+}\left(x^{\tan(x)}\right) = \lim_{x\to 0^+}\left(e^{\ln\left(x^{\tan(x)}\right)}\right) = e^0 = 1$$

$$\lim_{x\to 0^+}\left(\ln\left(x^{\tan(x)}\right)\right) = \lim_{x\to 0^+}\left(\tan(x)\ln(x)\right) = \lim_{x\to 0^+}\left(\frac{\ln(x)}{\cot(x)}\right)$$

$$= \lim_{x\to 0^+}\left(\frac{\frac{1}{x}}{-\csc^2(x)}\right) = \lim_{x\to 0^+}\left(\frac{-\sin^2(x)}{x}\right) = \lim_{x\to 0^+}\left(\frac{-2\sin(x)\cos(x)}{1}\right) = 0$$