Logarithmic Functions – $f(x) = \log_a(x)$, $a > 0$, $a \neq 1$

$\log_a(x) = y$ iff $a^y = x$ — inverse of exponential

Laws of Logarithms – $x, y > 0$, $r \in \mathbb{R}$

$\log_a(1) = 0$ — $\log_a(1) = x \rightarrow a^x = 1$

$\log_a(a) = 1$ — $\log_a(a) = x \rightarrow a^x = a$

$\log_a(a^r) = r$ — $\log_a(a^r) = x \rightarrow a^x = a^r$

$a^{\log_a(r)} = r$ — $a^x a^y = a^{x+y}$ — $a^{\log_a(r)} = x \rightarrow \log_a(x) = \log_a(r)$

$\log_a(xy) = \log_a(x) + \log_a(y)$ — $\log_a(xy) = M \rightarrow a^M = xy = a^{\log_a(x)} a^{\log_a(y)} = a^{\log_a(x) + \log_a(y)}$

$\log_a\left(\frac{x}{y}\right) = \log_a(xy^{-1}) = \log_a(x) + \log_a(y^{-1}) = \log_a(x) - \log_a(y)$

$\log_a(x^r) = \log_a(\underbrace{x \cdot x \cdot x \cdots x}_{r \text{ times}}) = \underbrace{\log_a(x) + \log_a(x) + \ldots + \log_a(x)}_{r \text{ times}} = r\log_a(x)$

Common Log

$\log_{10}(x) = \log(x) = y$

$10^y = x$

Natural Log

$\log_e(x) = \ln(x) = y$

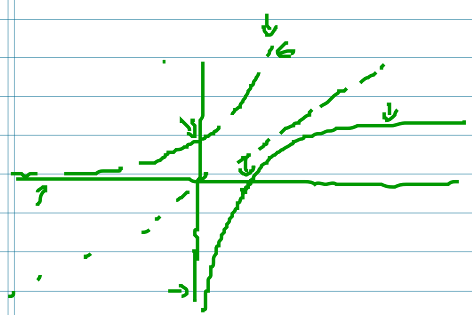
$e^y = x$

$\ln(e^x) = x$, $\forall x \in \mathbb{R}$

$e^{\ln(x)} = x$, $\forall x > 0$

$\ln(e) = 1$

$\ln(1) = 0$



$a > 1$

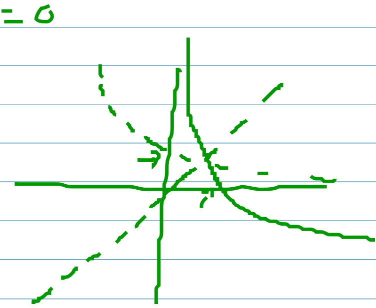
D: $(0, \infty)$

R: $\mathbb{R}$

x-int: $(1, 0)$

y-int: None

VA: $x = 0$



$0 < a < 1$

Logarithmic Functions — $f(x)=\log_a(x)$, $a>0$, $a\neq 1$

$\log_a(x)=y$ iff $a^y=x$ — inverse of exponential

Laws of Logarithms — $x,y>0$, $r\in\mathbb{R}$

$\log_a(1)=0$ — $\log_a(1)=x \Rightarrow a^x=1$

$\log_a(a)=1$ — $\log_a(a)=x \Rightarrow a^x=a$

$\Rightarrow$ $\log_a(a^r)=r$ — $\log_a(a^r)=x \Rightarrow a^x=a^r$

$\rightarrow$ $a^{\log_a(r)}=r$ — $a^xa^y=a^{x+y}$ — $a^{\log_a(r)}=x \Rightarrow \log_a(x)=\log_a(r)$

$\Rightarrow$ $\log_a(xy)=\log_a(x)+\log_a(y)$ — $\log_a(xy)=M \Rightarrow a^{M}=xy$

$$=a^{\log_a(x)}a^{\log_a(y)}=a^{\log_a(x)+\log_a(y)}$$

$\rightarrow$ $\log_a\left(\frac{x}{y}\right)=\log_a(xy^{-1})=\log_a(x)+\log_a(y^{-1})=\log_a(x)-\log_a(y)$

$$\log_a(x^r)=\log_a(\underbrace{x\cdot x\cdot x\cdots x}_{r\text{ times}})=\underbrace{\log_a(x)+\log_a(x)+\dots+\log_a(x)}_{r\text{ times}}=r\log_a(x)$$

Common Log

$\log_{10}(x)=\log(x)=y$

$10^y=x$

Natural Log

$\log_e(x)=\ln(x)=y$

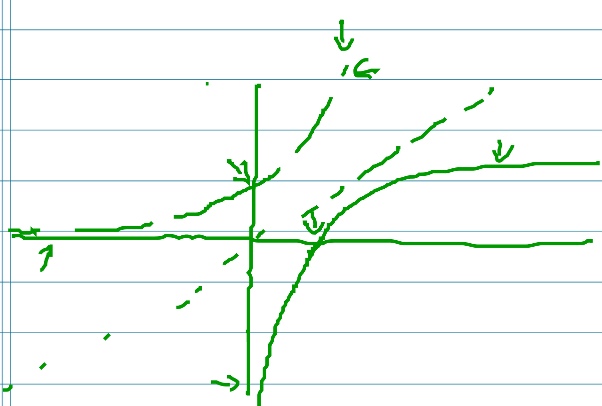
$e^y=x$

$\ln(e^x)=x$, $\forall x\in\mathbb{R}$

$e^{\ln(x)}=x$, $\forall x>0$

$\ln(e)=1$

$\ln(1)=0$



D: $(0,\infty)$

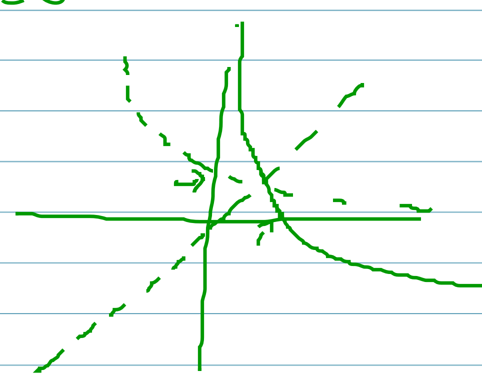
R: $\mathbb{R}$

x-int: $(1,0)$

y-int: None

VA: $x=0$

$a>1$



$0<a<1$

Change of Base

$u = v$, ~~then~~ iff $\log_a(u) = \log$

$2^x = 5$

$\log(2^x) = \log(5)$ $\qquad$ $\log_2(2^x) = \log_2(5)$ $\qquad$ $\ln(2^x) = \ln(5)$

$x\log(2) = \log(5)$ $\qquad$ $x = \log_2(5)$ $\qquad$ $x\ln(2) =$

$x = \dfrac{\log(5)}{\log(2)} \neq \dfrac{5}{2}$ $\quad 2.32$ $\qquad$ $x = \dfrac{\ln(5)}{\ln(2)} \to 2.32$

$$\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$$

$$\log_7(5) = \frac{\log(5)}{\log(7)} = \frac{\ln(5)}{\ln(7)} = \frac{\log_{21}(5)}{\log_{21}(7)}$$

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$

$\log_a(M) = \dfrac{\log_b(M)}{\log_b(a)}$ $\qquad \to \log_a\left(\dfrac{x}{y}\right) = \log_a(x) - \log_a(y)$

$\log_a(x \pm y) \neq \log_a(x) \pm \log_a(y)$

$\log_a(xy^n) \neq n\log_a(x \cdot y) = \log_a((xy)^n)$

$\ln(96) = \ln(2^5 \cdot 3) = \ln(2^5) + \ln(3)$

$= 5\ln(2) + \ln(3)$

$= 5(0.6931) + 1.0986$

$= 3.4655 + 1.0986$

$= \underline{4.5641} \to \underline{4.5643}$

96: 2, 48; 2, 24; 4, 6; 2, 2, 2, 3

$\ln(2) = 0.6931$

$\ln(3) = 1.0986$

$\ln(5) = 1.6094$

$\ln(7) = 1.9459$

$\ln\left(\dfrac{x^2y^5}{w^3z^8}\right) = \ln(x^2y^5) - \ln(w^3z^8)$

$= \ln(x^2) + \ln(y^5) - (\ln(w^3) + \ln(z^8))$

$= 2\ln(x) + 5\ln(y) - 3\ln(w) - 8\ln(z)$