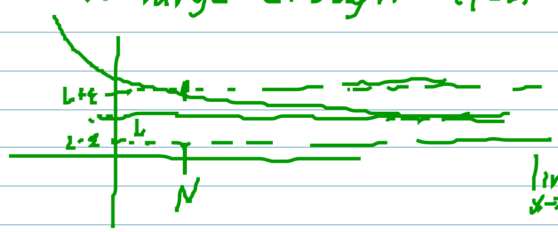
<u>Infinite Limit</u> – $\lim_{x\to a} (f(x)) = \pm\infty$

<u>Vertical Asymptote</u> – $x = a$

$\lim_{x\to a^+} (f(x)) = \pm\infty$ $\quad \lim_{x\to a^-} (f(x)) = \pm\infty$ $\quad \lim_{x\to a} (f(x)) = \pm\infty$

<u>Limit at Infinity</u> – $\lim_{x\to\infty} (f(x)) = L$ OR $\lim_{x\to-\infty} (f(x)) = L$

- f is defined on (a,∞) OR $(-\infty, a)$ for some $a \in \mathbb{R}$
- can get as close to L as you want by making x large enough (positive OR negative)



Horizontal Asymptote
$y = L$

$\lim_{x\to\infty} (f(x)) = L$ OR $\lim_{x\to-\infty} (f(x)) = L$

<u>Infinite Limit at Infinity</u> – $\lim_{x\to-\infty} (f(x)) = \pm\infty$ OR $\lim_{x\to\infty} (f(x)) = \pm\infty$

- $\infty - \infty$ is indeterminant

<u>ε-δ definitions</u>

$\lim_{x\to a} (f(x)) = \infty$ Given $M > 0$, $\exists\, \delta > 0$ st if $|x-a| < \delta$, then $f(x) > M$

$\lim_{x\to\infty} (f(x)) = L$ Given $\varepsilon > 0$, $\exists N$ st $\forall x > N$, $|f(x) - L| < \varepsilon$

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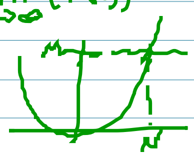

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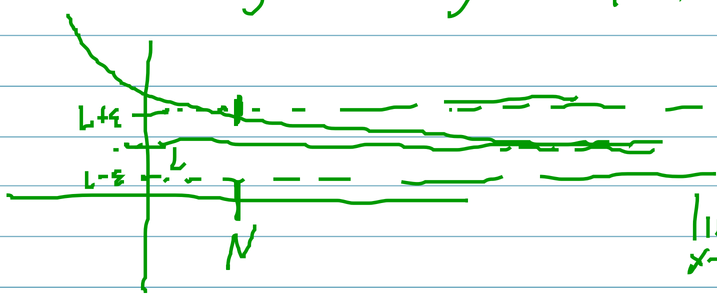

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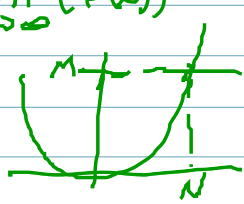


$$\lim_{x\to\infty}\left(\frac{x^2+5}{x^2-3}\right) = \lim_{x\to\infty}\left(\frac{\cancel{x^2}\left(1+\frac{5}{x^2}\right)}{\cancel{x^2}\left(1-\frac{3}{x^2}\right)}\right) = \frac{1+0}{1-0} = 1$$

$$\lim_{x\to\infty}\left(\frac{7x^4+5x^2-3x}{-2x^4+7x^3-8x^2+9}\right) = \lim_{x\to\infty}\left(\frac{\cancel{x^4}\left(7+\frac{5}{x^2}-\frac{3}{x^3}\right)}{\cancel{x^4}\left(-2+\frac{7}{x}-\frac{8}{x^2}+\frac{9}{x^4}\right)}\right) = -\frac{7}{2}$$

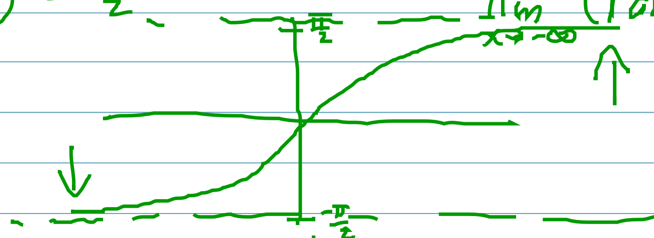
$$\lim_{x\to-\infty}\left(\frac{5x^2-3x+7}{-4x^3+9x^2-8}\right) = \lim_{x\to-\infty}\left(\frac{\cancel{x^2}\left(5-\frac{3}{x}+\frac{7}{x^2}\right)}{\cancel{x^2}\left(-4x+9-\frac{8}{x}\right)}\right) = 0$$

Theorem

If $r>0$, $r\in\mathbb{Q}$, then $\lim_{x\to\infty}\left(\frac{1}{x^r}\right)=0$

If $r>0$, $r\in\mathbb{Q}$, x^r is defined for all x, then $\lim_{x\to-\infty}\left(\frac{1}{x^r}\right)=0$

$\lim_{x\to\infty}\left(\tan^{-1}(x)\right) = \frac{\pi}{2}$ $\qquad \lim_{x\to-\infty}\left(\tan^{-1}(x)\right) = -\frac{\pi}{2}$



$$\lim_{x\to\infty}\left(\frac{\sqrt{2x^2+1}}{3x-5}\right) = \lim_{x\to\infty}\left(\frac{\sqrt{x^2\left(2+\frac{1}{x^2}\right)}}{x\left(3-\frac{5}{x}\right)}\right) = \lim_{x\to\infty}\left(\frac{\cancel{|x|}\sqrt{2+\frac{1}{x^2}}}{\cancel{x}\left(3-\frac{5}{x}\right)}\right) = \frac{\sqrt{2}}{3}$$

$$\lim_{x\to-\infty}\left(\frac{\sqrt{2x^2+1}}{3x-5}\right) = \lim_{x\to-\infty}\left(\frac{\sqrt{x^2\left(2+\frac{1}{x^2}\right)}}{x\left(3-\frac{5}{x}\right)}\right) = \lim_{x\to-\infty}\left(\frac{|x|\sqrt{2+\frac{1}{x^2}}}{x\left(3-\frac{5}{x}\right)}\right)$$

$$= -\frac{\sqrt{2}}{3}$$

$$\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x} = \lim_{x\to\infty}\left(\frac{|x^2+1|-x^2}{\sqrt{x^2+1}+x}\right) = \lim_{x\to\infty}\left(\frac{1}{\sqrt{x^2+1}+x}\right)$$

$$= 0$$

$$\lim_{x\to\infty}\left(x^3-x\right) = \lim_{x\to\infty}\left(x\left(x^2-1\right)\right) = (\infty)(\infty) = \infty$$