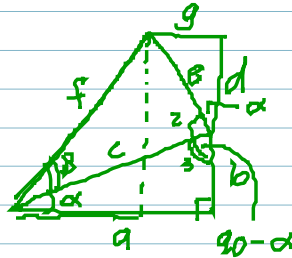


$$\sin(\alpha + \beta) = \frac{b+d}{f}$$

$$= \frac{b}{f} + \frac{d}{f}$$

$$= \frac{b}{f} \cdot \frac{c}{c} + \frac{d}{f} \cdot \frac{e}{e}$$

$$= \frac{b}{f} \cdot \frac{c}{f} + \frac{d}{f} \cdot \frac{e}{f}$$



$$\sin(\alpha) = \frac{g}{f} = \frac{g}{f}$$

$$\sin(\beta) = \frac{d}{f}$$

$$\cos(\alpha) = \frac{c}{f} = \frac{c}{f}$$

$$\cos(\beta) = \frac{e}{f}$$

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha + (-\beta))$$

$$= \sin(\alpha) \cos(-\beta) + \cos(\alpha) \sin(-\beta)$$

$$= \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$$

$$\rightarrow \sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha + \beta) = \frac{a-g}{f} = \frac{a}{f} - \frac{g}{f} = \frac{a}{f} \cdot \frac{c}{c} - \frac{g}{f} \cdot \frac{e}{e} = \frac{a}{f} \cdot \frac{c}{f} - \frac{g}{f} \cdot \frac{e}{f}$$

$$= \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha + (-\beta)) = \cos(\alpha) \cos(-\beta) - \sin(\alpha) \sin(-\beta)$$

$$= \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$$

$$\rightarrow \cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\sin(\alpha \pm \beta)}{\cos(\alpha \pm \beta)} = \frac{\sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)}{\cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)}$$

$$= \frac{\sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)}{\left(1 \mp \frac{\sin(\alpha) \sin(\beta)}{\cos(\alpha) \cos(\beta)}\right)} \left( \frac{1}{\cos(\alpha) \cos(\beta)} \right)$$

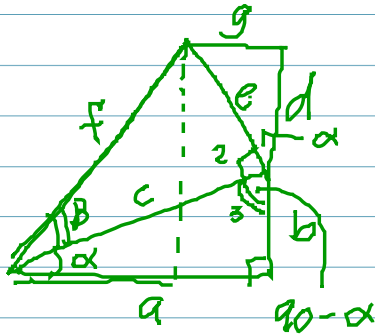
$$= \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)}$$

$$\sin(\alpha + \beta) = \frac{b+d}{f}$$

$$= \frac{b}{f} + \frac{d}{f}$$

$$= \frac{b}{f} \cdot \frac{c}{c} + \frac{d}{f} \cdot \frac{e}{e}$$

$$= \frac{b}{c} \cdot \frac{c}{f} + \frac{d}{e} \cdot \frac{e}{f}$$



$$\sin(\alpha) = \frac{b}{c} = \frac{g}{f}$$

$$\sin(\beta) = \frac{d}{c}$$

$$\cos(\alpha) = \frac{a}{c} = \frac{g}{f}$$

$$\cos(\beta) = \frac{e}{c}$$

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha + (-\beta))$$

$$= \sin(\alpha) \cos(-\beta) + \cos(\alpha) \sin(-\beta)$$

$$= \sin(\alpha) \cos(\beta) - \cos(\alpha) \sin(\beta)$$

$$\rightarrow \sin(\alpha \pm \beta) = \sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)$$

$$\cos(\alpha + \beta) = \frac{a-g}{f} = \frac{a}{f} - \frac{g}{f} = \frac{a}{f} \cdot \frac{c}{c} - \frac{g}{f} \cdot \frac{e}{e} = \frac{a}{c} \cdot \frac{c}{f} - \frac{g}{e} \cdot \frac{e}{f}$$

$$= \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha + (-\beta)) = \cos(\alpha) \cos(-\beta) - \sin(\alpha) \sin(-\beta)$$

$$= \cos(\alpha) \cos(\beta) + \sin(\alpha) \sin(\beta)$$

$$\rightarrow \cos(\alpha \pm \beta) = \cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\sin(\alpha \pm \beta)}{\cos(\alpha \pm \beta)} = \frac{\sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)}{\cos(\alpha) \cos(\beta) \mp \sin(\alpha) \sin(\beta)}$$

$$= \frac{\sin(\alpha) \cos(\beta) \pm \cos(\alpha) \sin(\beta)}{\left(1 \mp \frac{\sin(\alpha) \sin(\beta)}{\cos(\alpha) \cos(\beta)}\right)} \left(\frac{1}{\cos(\alpha) \cos(\beta)}\right)$$

$$= \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha) \tan(\beta)}$$

$$\begin{aligned} \sin(15^\circ) &= \sin(60^\circ - 45^\circ) = \sin(60^\circ)\cos(45^\circ) - \cos(60^\circ)\sin(45^\circ) \\ &\rightarrow = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \sin(15^\circ) &= \sin(45^\circ - 30^\circ) = \sin(45^\circ)\cos(30^\circ) - \cos(45^\circ)\sin(30^\circ) \\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{6} - \sqrt{2}}{4} \end{aligned}$$

$$\begin{aligned} \cos(165^\circ) &= \cos(120^\circ + 45^\circ) && \begin{matrix} 30, 45, 60, 90 \\ 120, 135, 150, 180 \end{matrix} \\ &= \cos(120^\circ)\cos(45^\circ) - \sin(120^\circ)\sin(45^\circ) \\ &= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{-\sqrt{2} - \sqrt{6}}{4} \end{aligned}$$

$$\begin{aligned} \sin\left(\frac{5\pi}{12}\right) &= \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right) && \begin{matrix} \frac{\pi}{6} & \frac{\pi}{4} & \frac{\pi}{3} & \frac{\pi}{2} \\ \frac{2\pi}{12} & \frac{3\pi}{12} & \frac{4\pi}{12} & \frac{6\pi}{12} \end{matrix} \\ &= \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right) \\ &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\ &= \frac{\sqrt{2} + \sqrt{6}}{4} \end{aligned}$$

$$\begin{aligned} \tan\left(\frac{\pi}{12}\right) &= \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)} \\ &= \frac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \end{aligned}$$

$$\begin{aligned} \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{5\pi}{6}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{5\pi}{6}\right) &= \sin\left(\frac{\pi}{3} - \frac{5\pi}{6}\right) \\ &= \sin\left(-\frac{3\pi}{6}\right) = \sin\left(-\frac{\pi}{2}\right) = -1 \end{aligned}$$

$$\sin(\alpha) = \frac{3}{5}, \alpha \text{ in Quad I} \quad \cos(\beta) = -\frac{5}{13} \quad \beta \text{ in Quad III}$$

$$\begin{aligned} \cos(\alpha + \beta) &= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta) \\ \cos\left(\frac{4}{5}\right)\cos\left(-\frac{5}{13}\right) &\rightarrow = \left(\frac{4}{5}\right)\left(-\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(-\frac{12}{13}\right) \\ &= \frac{-20 + 36}{65} = \frac{16}{65} \end{aligned}$$

