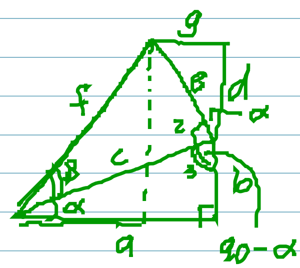
$$\sin(\alpha+\beta) = \frac{b+d}{f}$$
$$= \frac{b}{f} + \frac{d}{f}$$
$$= \frac{b}{f}\cdot\frac{c}{c} + \frac{d}{f}\cdot\frac{e}{e}$$
$$= \frac{b}{c}\cdot\frac{c}{f} + \frac{d}{e}\cdot\frac{e}{f}$$



$\sin(\alpha) = \frac{b}{c} = \frac{g}{e}$
$\sin(\beta) = \frac{e}{f}$
$\cos(\alpha) = \frac{a}{c} = \frac{d}{e}$
$\cos(\beta) = \frac{c}{f}$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha-\beta) = \sin(\alpha+(-\beta))$$
$$= \sin(\alpha)\cos(-\beta) + \cos(\alpha)\sin(-\beta)$$
$$= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\rightarrow \sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha+\beta) = \frac{a-g}{f} = \frac{a}{f} - \frac{g}{f} = \frac{a}{f}\frac{c}{c} - \frac{g}{f}\frac{e}{e} = \frac{a}{c}\cdot\frac{c}{f} - \frac{g}{e}\cdot\frac{e}{f}$$
$$= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha-\beta) = \cos(\alpha+(-\beta)) = \cos(\alpha)\cos(-\beta) - \sin(\alpha)\sin(-\beta)$$
$$= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\rightarrow \cos(\alpha \pm \beta)\ \ \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha \pm \beta) = \frac{\sin(\alpha\pm\beta)}{\cos(\alpha\pm\beta)} = \frac{\sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)}$$

$$= \frac{\sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)}{\left(1 \mp \frac{\sin(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)}\right)} \leftarrow \left(\frac{1}{\cos(\alpha)\cos(\beta)}\right)$$

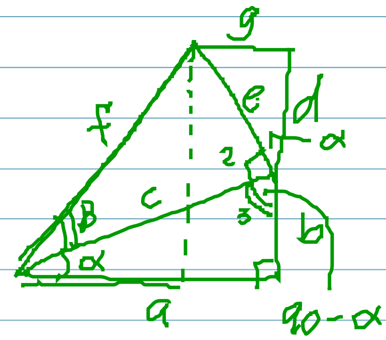
$$= \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$$

$$\sin(\alpha+\beta) = \frac{b+d}{f}$$

$$= \frac{b}{f} + \frac{d}{f}$$

$$= \frac{b}{f}\cdot\frac{c}{c} + \frac{d}{f}\cdot\frac{e}{e}$$

$$= \underline{\frac{b}{c}}\cdot\underline{\frac{c}{f}} + \underline{\frac{d}{e}}\cdot\underline{\frac{e}{f}}$$



$$\sin(\alpha) = \frac{b}{c} = \frac{g}{e}$$
$$\sin(\beta) = \frac{e}{f}$$
$$\cos(\alpha) = \frac{a}{c} = \frac{d}{e}$$
$$\cos(\beta) = \frac{c}{f}$$

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha-\beta) = \sin(\alpha+(-\beta))$$
$$= \sin(\alpha)\cos(-\beta) + \cos(\alpha)\sin(-\beta)$$
$$= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\rightarrow \sin(\alpha\pm\beta) = \underline{\sin(\alpha)\cos(\beta)} \pm \underline{\cos(\alpha)\sin(\beta)}$$

$$\cos(\alpha+\beta) = \frac{a-g}{f} = \frac{a}{f} - \frac{g}{f} = \frac{a}{f}\frac{c}{c} - \frac{g}{f}\frac{e}{e} = \frac{a}{c}\cdot\frac{c}{f} - \frac{g}{e}\cdot\frac{e}{f}$$
$$= \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha-\beta) = \cos(\alpha+(-\beta)) = \cos(\alpha)\cos(-\beta) - \sin(\alpha)\sin(-\beta)$$
$$= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\rightarrow \cos(\alpha\pm\beta) \quad \underline{\cos(\alpha)\cos(\beta)} \mp \underline{\sin(\alpha)\sin(\beta)}$$

$$\tan(\alpha\pm\beta) = \frac{\sin(\alpha\pm\beta)}{\cos(\alpha\pm\beta)} = \frac{\sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)}{\underline{\cos(\alpha)\cos(\beta)} \mp \sin(\alpha)\sin(\beta)}$$

$$= \frac{\sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)}{\left(1 \mp \frac{\sin(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)}\right)} \Leftarrow \left(\frac{1}{\cos(\alpha)\cos(\beta)}\right)$$

$$= \frac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$$

$$\sin(15^\circ) = \sin(60^\circ - 45^\circ) = \sin(60^\circ)\cos(45^\circ) - \cos(60^\circ)\sin(45^\circ)$$

$$\rightarrow = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\sin(15^\circ) = \sin(45^\circ - 30^\circ) = \sin(45^\circ)\cos(30^\circ) - \cos(45^\circ)\sin(30^\circ)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

30, 45, 60, 90
120, 135, 150, 180

$$\cos(165^\circ) = \cos(120^\circ + 45^\circ)$$

$$= \cos(120^\circ)\cos(45^\circ) - \sin(120^\circ)\sin(45^\circ)$$

$$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{-\sqrt{2} - \sqrt{6}}{4}$$

$\frac{\pi}{6}$ $\frac{\pi}{4}$ $\frac{\pi}{3}$ $\frac{\pi}{2}$

$\frac{2\pi}{12}$ $\frac{3\pi}{12}$ $\frac{4\pi}{12}$ $\frac{6\pi}{12}$

$$\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$$

$$= \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)}$$

$$= \frac{3}{3} \cdot \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)\left(\frac{\sqrt{3}}{3}\right)} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

$$\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{5\pi}{6}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{2\pi}{6} - \frac{5\pi}{6}\right)$$

$$= \sin\left(\frac{-3\pi}{6}\right) = \sin\left(-\frac{\pi}{2}\right) = -1$$

$\sin(\alpha) = \frac{3}{5}$, α in Quad I $\qquad \cos(\beta) = -\frac{5}{13}$ β in Quad III

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cancel{\cos\left(\tfrac{4}{5}\right)\cos\left(\tfrac{-5}{13}\right)} \rightarrow = \left(\frac{4}{5}\right)\left(\frac{-5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right)$$

$$= \frac{-20 + 36}{65} = \frac{16}{65}$$