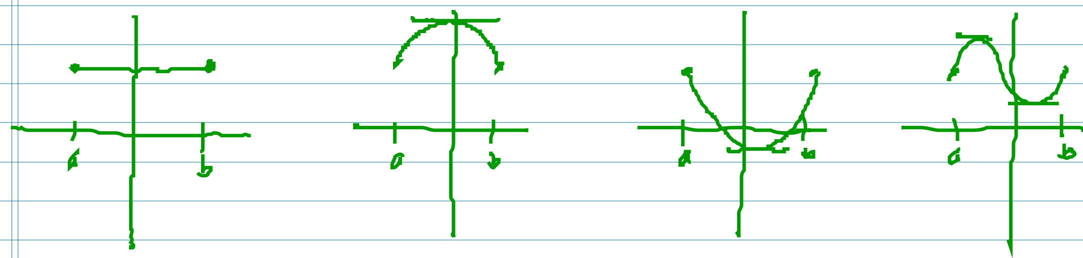
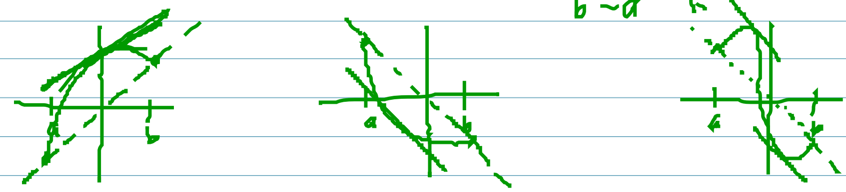
Rolle's Theorem

If f: continuous on $[a,b]$, differentiable on (a,b), $f(a)=f(b)$, Then $\exists\, c \in (a,b)$ st $f'(c)=0$



Mean Value Theorem

If f: continuous on $[a,b]$, differentiable on (a,b), Then $\exists\, c \in (a,b)$ st $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ OR $f'(c)(b-a) = f(b)-f(a)$



Theorem

If $f'(x)=0$, $\forall x \in (a,b)$, Then $f(x) = K = \text{constant}$, $\forall x \in (a,b)$

Corollary

If $f'(x) = g'(x)$, $\forall x \in (a,b)$, Then $(f-g)(x) = K = \text{constant}$, $\forall x \in (a,b)$

OR $f(x) = g(x) + K$, $\forall x \in (a,b)$, $K = \text{constant}$

Fixed Point – a st $f(a)=a$

$f(x) = x^3 + x - 8$ $\quad [-1,3]$

$f(-1) = (-1)^3 + (-1) - 8 = -10$

$f(3) = 3^3 + 3 - 8 = 22$

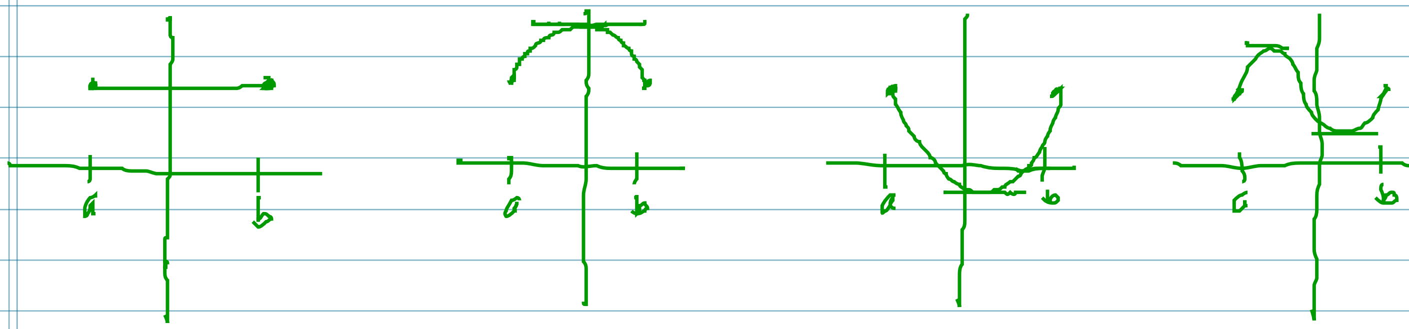
$m_{sec} = \dfrac{f(3)-f(-1)}{3-(-1)} = \dfrac{22-(-10)}{3-(-1)} = \dfrac{32}{4} = 8$

$f'(x) = 3x^2 - 1$ $\quad 3x^2 - 1 = 8$ $\quad x^2 = 3$

$3x^2 = 9$ $\quad x = \pm\sqrt{3} \rightarrow x = \sqrt{3}$

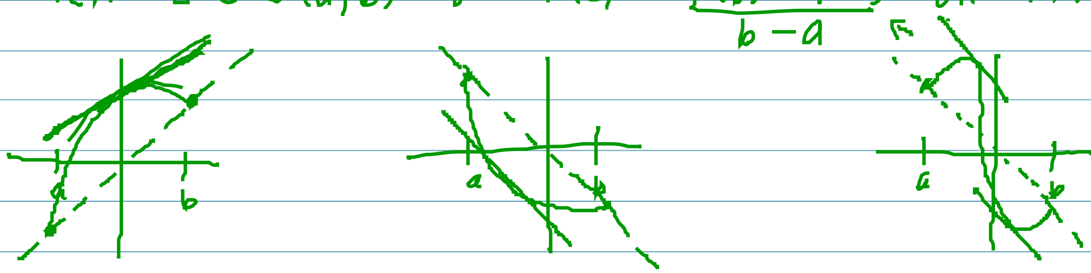
Rolle's Theorem

If f = continuous on $[a,b]$, differentiable on (a,b), $f(a) = f(b)$, Then $\exists\, c \in (a,b)$ st $f'(c) = 0$



Mean Value Theorem

If f = continuous on $[a,b]$, differentiable on (a,b), Then $\exists\, c \in (a,b)$ st $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ OR $f'(c)(b-a) = f(b)-f(a)$



Theorem

If $f'(x) = 0$, $\forall x \in (a,b)$, Then $f(x) = K$ = constant, $\forall x \in (a,b)$

Corollary

If $f'(x) = g'(x)$, $\forall x \in (a,b)$, Then $(f-g)(x) = K$ = constant, $\forall x \in (a,b)$
or $f(x) = g(x) + K$, $\forall x \in (a,b)$, K = constant

Fixed Point - a st $f(a) = a$

$f(x) = x^3 + x - 8$ $\quad [-1, 3]$ $\quad m_{sec} = \dfrac{f(3)-f(-1)}{3-(-1)}$

$f(-1) = (-1)^3 + (-1) - 8 = -10$

$f(3) = 3^3 + 3 - 8 = 22$ $\quad = \dfrac{22-(-10)}{3-(-1)} = \dfrac{32}{4} = 8$

$f'(x) = 3x^2 - 1$ $\quad 3x^2 - 1 = 8$ $\quad x^2 = 3$

$3x^2 = 9$ $\quad x = \pm\sqrt{3} \rightarrow x = \sqrt{3}$

$f(x) = x^2 - 4x + 5$ $\quad [-1, 5]$

$f(5) = 5^2 - 4(5) + 5 = 10$

$f(-1) = (-1)^2 - 4(-1) + 5 = 10$ $\quad m_{sec} = \dfrac{f(5) - f(-1)}{5 - (-1)} = \dfrac{10 - 10}{5 + 1} = \dfrac{0}{6} = 0$

$f'(x) = 2x - 4$ $\quad 2x - 4 = 0$

$x = 2$

<u>Proof of Rolle's Theorem</u>

case 1: $f(x) = k = \text{constant}$

$f'(x) = 0, \ \forall x \in (a, b)$, so pick any $c \in (a, b)$

case 2: $\exists x \in (a, b)$ st $f(x) > f(a)$

$\exists c \in [a, b]$ st $f(c) =$ ~~Local~~ Absolute Maximum

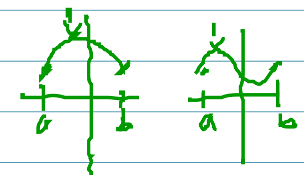
(Extreme Value Theorem)

$f(a) = f(b)$, so $c \in (a, b)$

$f(c) =$ Local Maximum

f = differentiable on (a, b) + $c \in (a, b)$

so $f'(c) = 0$ (Fermat's Theorem)



case 3: $\exists x \in (a, b)$ st $f(x) < f(a)$

$\exists c \in [a, b]$ st $f(c) =$ Absolute Minimum

(Extreme Value Theorem)

$f(a) = f(b)$, so $c \in (a, b)$

$f(c) =$ Local Minimum

f = differentiable on (a, b) + $c \in (a, b)$

so $f'(c) = 0$

(Fermat's Theorem)