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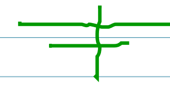
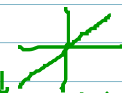
Constant Function – $\frac{d}{dx}(c) = 0$



Power Functions – $\frac{d}{dx}x = 1$



$f(x)=x^2$ $\quad \lim_{h\to 0}\left(\frac{(x+h)^2-x^2}{h}\right) = \lim_{h\to 0}\left(\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}\right) = \lim_{h\to 0}\left(\frac{2xh+h^2}{h}\right)$

$= \lim_{h\to 0}(2x+h) = 2x \leftarrow$

$f(x)=x^3$ $\quad \lim_{h\to 0}\left(\frac{(x+h)^3-x^3}{h}\right) = \lim_{h\to 0}\left(\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}\right)$

$\lim_{h\to 0}\left(\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}\right) = 3x^2 \leftarrow$

$f(x)=x^n$ $\quad \lim_{h\to 0}\left(\frac{(x+h)^n-x^n}{h}\right) = \lim_{h\to 0}\left(\frac{\cancel{x^n}+nx^{n-1}h+\dots+h^n-\cancel{x^n}}{h}\right)$

$\lim_{h\to 0}\left(\frac{\cancel{h}(nx^{n-1}+\dots+h^{n-1})}{\cancel{h}}\right) = nx^{n-1} \leftarrow$

Power Rule – $\frac{d}{dx}x^n = nx^{n-1}$ $\quad n\in\mathbb{R}$ $\quad \sqrt{x}^{\,n-1}$

Constant Multiple Rule – $\frac{d}{dx}(c\,f(x)) = c\,\frac{d}{dx}f(x)$

$\lim_{h\to 0}\left(\frac{c\,f(x+h)-c\,f(x)}{h}\right) = \lim_{h\to 0}\left(c\left(\frac{f(x+h)-f(x)}{h}\right)\right) = c\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$

$= c\,f'(x)$

Sum/Difference Rule – $\frac{d}{dx}(f(x)\pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

$\lim_{h\to 0}\left(\frac{f(x+h)\pm g(x+h)-(f(x)\pm g(x))}{h}\right) = \lim_{h\to 0}\left(\frac{(f(x+h)-f(x))\pm(g(x+h)-g(x))}{h}\right)$

$= \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right) \pm \lim_{h\to 0}\left(\frac{g(x+h)-g(x)}{h}\right) = f'(x)\pm g'(x)$

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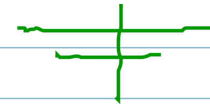
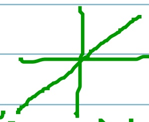
Constant Function – $\frac{d}{dx}(c) = 0$



Power Functions – $\frac{d}{dx}x = 1$



$f(x) = x^2$ $\quad \lim_{h\to 0}\left(\frac{(x+h)^2 - x^2}{h}\right) = \lim_{h\to 0}\left(\frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}\right) = \lim_{h\to 0}\left(\frac{2xh+h^2}{h}\right)$

$= \lim_{h\to 0}(2x + h) = 2x \Leftarrow$

$f(x) = x^3$ $\quad \lim_{h\to 0}\left(\frac{(x+h)^3 - x^3}{h}\right) = \lim_{h\to 0}\left(\frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}\right)$

$\lim_{h\to 0}\left(\frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}\right) = 3x^2 \Leftarrow$

$f(x) = x^n$ $\quad \lim_{h\to 0}\left(\frac{(x+h)^n - x^n}{h}\right) = \lim_{h\to 0}\left(\frac{\cancel{x^n} + nx^{n-1}h + \ldots + h^n - \cancel{x^n}}{h}\right)$

$\lim_{h\to 0}\left(\frac{\cancel{h}(nx^{n-1} + \ldots + h^{n-1})}{\cancel{h}}\right) = nx^{n-1} \Leftarrow$

Power Rule – $\frac{d}{dx}x^n = nx^{n-1}$ $\quad \sqrt{x}^{\,n-1}$

$n \in \mathbb{R}$

Constant Multiple Rule – $\frac{d}{dx}(c\,f(x)) = c\,\frac{d}{dx}f(x)$

$\lim_{h\to 0}\left(\frac{c\,f(x+h) - c\,f(x)}{h}\right) = \lim_{h\to 0}\left(c\left(\frac{f(x+h)-f(x)}{h}\right)\right) = c\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$

$= c\,f'(x)$

Sum/Difference Rule – $\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

$\lim_{h\to 0}\left(\frac{f(x+h) \pm g(x+h) - (f(x) \pm g(x))}{h}\right) = \lim_{h\to 0}\left(\frac{(f(x+h) - f(x)) \pm (g(x+h) - g(x))}{h}\right)$

$= \lim_{h\to 0}\left(\frac{f(x+h) - f(x)}{h}\right) \pm \lim_{h\to 0}\left(\frac{g(x+h) - g(x)}{h}\right) = f'(x) \pm g'(x)$

Exponential Functions — $f(x) = b^x$

$$f'(x) = \lim_{h\to 0}\left(\frac{b^{x+h} - b^x}{h}\right) = \lim_{h\to 0}\left(\frac{b^x b^h - b^x}{h}\right) = \lim_{h\to 0}\left(\frac{b^x(b^h - 1)}{h}\right)$$

$$= b^x \underbrace{\lim_{h\to 0}\left(\frac{b^h - 1}{h}\right)}_{f'(0)} = \underbrace{f'(0)}_{\ln(b)}\, b^x$$

$$\frac{d}{dx} b^x = \ln(b)\, b^x$$

Euler's Number — e

$$\lim_{h\to 0}\left(\frac{e^h - 1}{h}\right) = 1$$

$f(x) = 2^x$ $\quad f'(x) = \ln(2)\, 2^x$ $\quad f'(0) = \ln(2) = 0.6931$ ←

$f(x) = 3^x$ $\quad f'(x) = \ln(3)\, 3^x$ $\quad f'(0) = \ln(3) = 1.0986$

$$\frac{d}{dx} e^x = e^x \to \underbrace{\ln(e)}_{1} e^x \to e^x \qquad e \approx 2.718$$

$$f(x) = 5x^7 + 3x^4 - 9x^2 + 2x - 6$$
$$f'(x) = 5\cdot 7x^6 + 3\cdot 4x^3 - 9\cdot 2x + 2 - 0$$
$$= 35x^6 + 12x^3 - 18x + 2$$

$$f(x) = \frac{5x^7 + 10x^3 - 4x^2 + 3x^{3/2}}{x^2} = 5x^5 + 10x - 4 + 3x^{-1/2}$$
$$f'(x) = 5\cdot 5x^4 + 10 - 0 + 3\left(-\tfrac{1}{2}\right) x^{-1/2 - 1}$$
$$= 25x^4 + 10 - \tfrac{3}{2} x^{-3/2}$$

$$f(x) = (x^5 - 2)(x^{-3} + 1) = x^2 + x^5 - 2x^{-3} - 2$$

$$f'(x) = 2x + 5x^4 - 2(-3)x^{-4} + 0 = 2x + 5x^4 + 6x^{-4}$$

$$f(x) = \frac{\sqrt{x} + x}{x^2} + 2^x - 4e^x = x^{-3/2} + x^{-1} + 2^x - 4e^x$$

$$f'(x) = -\tfrac{3}{2} x^{-5/2} - 1x^{-2} + \ln(2)\, 2^x - 4e^x$$