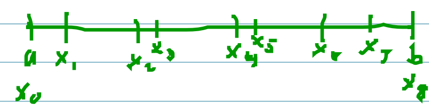
Partition – divide interval into subintervals

$a = x_0 < x_1 < x_2 < \ldots < x_n = b$

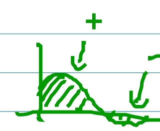
Δx_i = length of $[x_{i-1}, x_i] = x_i - x_{i-1}$

$x_i^* \in [x_{i-1}, x_i]$

$P = [x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n]$



Riemann Sums – $R_n = \sum_{i=1}^{n} f(x_i^*)\Delta x_i$



Definite Integral

$$\int_a^b f(x)\,dx = \lim_{\max \Delta x_i \to 0} \left(\sum_{i=1}^{n} f(x_i^*)\Delta x_i \right)$$

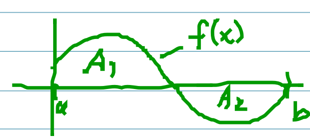
integrand

Theorem
If f = continuous on [a,b], or f has finite number of jump discontinuities on [a,b], then f = integrable on [a,b]
– $\int_a^b f(x)\,dx$ exists

Theorem
If f = integrable on [a,b], then $\int_a^b f(x)dx = \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(x_i)\Delta x \right)$,

$\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

Net Area



$A = A_1 - A_2 = \int_a^b f(x)dx$

signed area

__Partition__ – divide interval into subintervals

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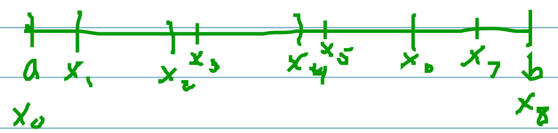

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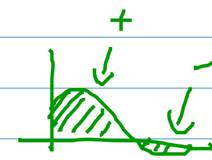

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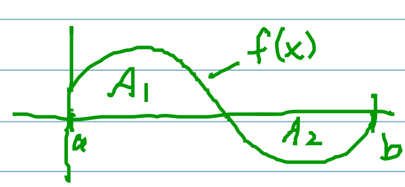
__Net Area__ .



$$A = A_1 - A_2 = \int_a^b f(x)\,dx$$

signed area

Properties of Summations

$$\sum_{i=1}^{n} c = \underbrace{c + c + c + c + \dots + c}_{n \text{ times}} = nc \quad , \quad c : \text{constant}$$

$$\sum_{i=1}^{n} c a_i = c a_1 + c a_2 + c a_3 + \dots + c a_n = c\,(a_1 + a_2 + \dots + a_n)$$

$$= c \sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n} (a_i \pm b_i) = a_1 \pm b_1 + a_2 \pm b_2 + \dots + a_n \pm b_n$$

$$= (a_1 + a_2 + \dots + a_n) \pm (b_1 + b_2 + \dots + b_n)$$

$$= \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$$

common summations

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(n+2)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$f(x) = x^2 + 2 \quad , \quad [-1, 1] \qquad \Delta x_n = \frac{1-(-1)}{n} = \frac{2}{n}$$

$$A = \lim_{n\to\infty} \left(\sum_{i=1}^{n} f(x_i)\, \Delta x_n \right) \qquad x_i = -1 + i\left(\frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} f\left(-1 + \frac{2i}{n}\right)\left(\frac{2}{n}\right)\right) = \lim_{n\to\infty} \left(\sum_{i=1}^{n} \left(\left(-1 + \frac{2i}{n}\right)^2 + 2\right) \frac{2}{n}\right)$$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \left(\left(1 - \frac{4i}{n} + \frac{4i^2}{n^2}\right) + 2\right) \frac{2}{n}\right) = \lim_{n\to\infty} \left(\sum_{i=1}^{n} \left(\frac{6}{n} - \frac{8i}{n^2} + \frac{8i^2}{n^3}\right)\right)$$

$$= \lim_{n\to\infty} \left(\sum_{i=1}^{n} \left(\frac{6}{n}\right) - \frac{8}{n^2} \sum_{i=1}^{n} i + \frac{8}{n^3} \sum_{i=1}^{n} i^2 \right) = \lim_{n\to\infty} \left(\frac{6}{n}(n) - \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{8}{n^3}\left(\frac{n(n+1)(n+2)}{6}\right)\right)$$

$$= \lim_{n\to\infty} \left(6 - 4\,\frac{n+1}{n} + \frac{4}{3}\,\frac{(n+1)(n+2)}{n} \right) = \lim_{n\to\infty} \left(6 - 4\left(1 + \frac{1}{n}\right) + \frac{4}{3}\left(1 + \frac{1}{n}\right)\left(1 + \frac{2}{n}\right)\right)$$

$$= 6 - 4(1+0) + \frac{4}{3}(1+0)(1+0) = 6 - 4 + \frac{4}{3} = \frac{10}{3}$$

Properties of Definite Integrals

$\int_a^b f(x)dx = -\int_b^a f(x)dx$ $\qquad \Delta x = \frac{b-a}{n} = \frac{a-b}{n}$

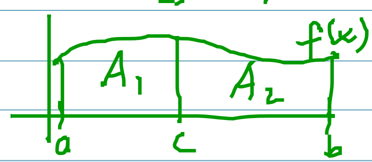
$\int_a^a f(x)dx = 0$ $\qquad \Delta x = \frac{a-a}{n} = 0$

$\int_a^b c\,dx = c(b-a)$

$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

$\int_a^b cf(x)dx = c\int_a^b f(x)dx$

$\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx \Leftarrow$



$a \le c \le b$, $\cancel{f(x) \ge 0}$

If $f(x) \ge 0, \forall x \in [a,b]$, then $\int_a^b f(x)dx \ge 0$

If $g(x) \ge f(x), \forall x \in [a,b]$, Then $\int_a^b g(x)dx \ge \int_a^b f(x)dx$

If $m \le f(x) \le M$, $\forall x \in [a,b]$, Then

$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$